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ABSTRACT

GRADES OR AGES: K-6. SUBJECT MATTER: Mathematics; sets and numbers. ORGANIZATION AND PHYSICAL APPEARANCE: The guide is divided into two sections--sets and numbers. Within each section the content is grouped into six levels in order of increasing difficulty. Each level contains from 3 to 15 concepts. Numerous diagrams and illustrations are included. The guide is offset printed with a soft cover. OBJECTIVES AND ACTIVITIES: For each concept presented, there is a brief statement of content and one or more behavioral objectives. Suggested activities are then listed for that concept. Activities considered to be of more than ordinary difficulty are marked with an asterisk. INSTRUCTIONAL MATERIALS: Materials needed for an activity are mentioned in the activity description. STUDENT ASSESSMENT: Student assessment is carried out through completion of the behavioral objectives listed with each concept. OPTIONS: The guide is suggestive only. No mention is made of timing or means of incorporating the concepts described into the total curriculum. (RT)

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MATHEMATICS Goals & Activities K-6

Part 1: Sets & Numbers

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Mathematics Goals and Activities: K-6 is a publication which identifies specific objectives of mathematics that are appropriate for elementary school children. It provides materials and experiences which take into account their individual levels of ability and achievement. The major purpose of this publication is to assist in the improvement of the teaching and learning of elementary mathematics in the schools of North Carolina.

A. Craig Phillips
State Superintendent of Public Instruction

ACKNOWLEDGEMENTS

More than 150 elementary teachers, supervisors and mathematics educators have had a direct part in the development of this publication. In the spring of 1968, an advisory committee consisting of teachers, supervisors, and college professors was formed to give direction to the overall curriculum project. Throughout the development of this publication this committee has made recommendations for which we are grateful.

The editing and much of the actual writing of this publication was done by Mr. John W. Ogle and Mr. Cleo M. Meek with assistance from Miss Nedra V. Mitchell, members of the Mathematics Division staff. Miss Jacqueline Garner did extensive writing for levels A through D. Mrs. Nina Clark, Mrs. Margaret King, Mrs. Clyde Phillips, Mrs. Olive Taylor, Mrs. Faith Thrift, and Mrs. Nancy Williams, and members of the North Carolina Kindergarten Association Publications Committee are teachers who also contributed greatly to the writing of the bulletin. We express sincere gratitude to them for their untiring efforts.

Dr. Edward Buffie, Dr. Edwina Deans, Dr. Mervin Keedy, Dr. Lola May, Dr. Eugene Nichols, and Dr. W. Gary Quast, nationally known mathematics educators and authors, served as consultants to the project and their assistance has been helpful in this curriculum effort.

In developing a publication of this nature, published resources are drawn upon as well as human resources. Materials from the following publishing companies were examined closely and many ideas included in them have been incorporated in some fashion in this publication: Addison-Wesley Publishing Company; American Book Company; Harcourt, Brace & World; Holt, Rinehart & Winston, Inc.; Houghton Mifflin Company; Laidlaw Brothers Publishers; Charles E. Merrill Publishing Company; William H. Sadlier, Inc.; Science Research Associates, Inc.; Scott Foresman & Company; Silver Burdett Company; and L. W. Singer & Company, Inc.

Without expert technical assistance, a publication of this scope would be impossible. The following State Department personnel played major roles in this project which are hereby gratefully acknowledged: Mr. Robert R. Jones, Mathematics Division Director, supervisory support and the handling of administrative details; Mr. William Chandler, editorial assistance; Mrs. Charlotte Barnes, suggestions for the kindergarten level of the material; Mrs. Helen Chu, preparation of the manuscript; Mrs. Patricia Bowers, illustrations and cover design; Mr. Steve Macon and Mrs. Sharon Newnam, illustrations; Mrs. Nyna Logan and Mrs. Mary S. Tyson, typing; and Mr. James Jackman, printing assistance.

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Jerome H. Melton
Assistant Superintendent for Program Services

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INTRODUCTION

In the spring of 1968, the Mathematics Division staff began planning for the development of a curriculum publication which would assist the teaching of mathematics in the elementary schools of North Carolina. To accomplish this purpose, the following tasks were identified:

(1) Identification of strands, or major topics, that permeate all of mathematics.

(2) Designation of the basic content normally taught at the various grade levels.

(3) Identification of specific objectives, written in behavioral terms which will enable teachers to evaluate more accurately each child's performance relating to these specific objectives.

(4) Development of activities for each objective so that the teaching of each objective can be done with vitality and meaning.

The mathematical content normally associated with grades K-6 has been identified within seven strands which are contained in the three parts of this publication.

Part 1 Sets and Numbers

Part 2 Operations and Mathematical Sentences

Part 3 Geometry, Measurement, and Graphs and Scale Drawings

These strands weave their way through elementary mathematics and provide the basis upon which it is built.

Within each strand specific content has been identified and then expressed as behavioral objectives with distinct pupil orientation. More than 600 of these objectives have been identified and are contained in the three volumes of this publication. They are arranged along a continuum which allows children to proceed at speeds consistent with their abilities. Furthermore, by working with objectives that identify observable outcomes, the pupil and the teacher can readily assess progress in terms of the stated objectives.

Realizing the difficulty many elementary teachers have in teaching mathematics successfully when they themselves have minimum preparation in the subject, the authors of this publication have provided teaching activities for each objective. These activities are merely suggested and may be used by different teachers in a variety of ways to stimulate dynamic, meaningful, and accurate teaching of mathematics.

Occasionally an activity is marked with an asterisk to indicate that it is an extension of the basic idea stated in the objective and because of this sophistication should be considered more optional than the other activities which precede it.

To use this publication locate your topic in the scope and sequence chart and then turn to the page indicated within the parentheses or turn to the index for the alphabetical listing of topics.

This publication can serve as a model to supplement and enrich local efforts in mathematics curriculum development. To be of real help to teachers, it should be adapted to local needs and resources. On each page following the activities, space is often provided for teachers to use in cross-referencing their textbooks and other materials with the content of the publication or to add other activities which they have found useful for helping students accomplish the objectives.

Scope & Sequence

How to Use the Scope and Sequence Chart

The content of this publication is outlined in the Scope and Sequence Chart. The pages which immediately follow contain the Chart for Sets and Numbers. At each level, A through G, you will find key ideas which form the nucleus of a sound, basic mathematics program.

The Scope and Sequence is presented in columns, an arrangement which makes this summary of the entire elementary mathematics program relatively easy to examine. In this sequence the teacher has ready access to material which immediately precedes and follows what the student is now studying. The numerals in parentheses following each topic of content refer to the page or pages of the text on which that particular content is found.

The content listed at any particular level is not intended to be the complete year's work for any particular student. It is arranged to present a natural development of Sets and Numbers. Knowledge of what has been presented at an earlier level is maintained and extended at subsequent levels.

SETS

SCOPE AND SEQUENCE CHART

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- Exposure to Sets (p. 10)
- Exploring with Sets (p. 12)
- Meaning of a Set (p. 13)
- Matching
  - One-to-one Correspondence (p. 14)
  - Comparison of Sets (p. 15)
- Ordering of Sets
  - The "one-more" Pattern (p. 16)
- Empty Set (p. 16)

Level B

- Meaning of a Set (p. 17)
- Matching
  - One-to-one Correspondence (p. 18)
  - Comparison of Sets (p. 19)
- Ordering of Sets
  - The "one-more" Pattern (p. 20)
- Empty Set (p. 21)
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Level C

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Level D

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Level E

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- Size of Sets (p. 35)
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Sets

Content

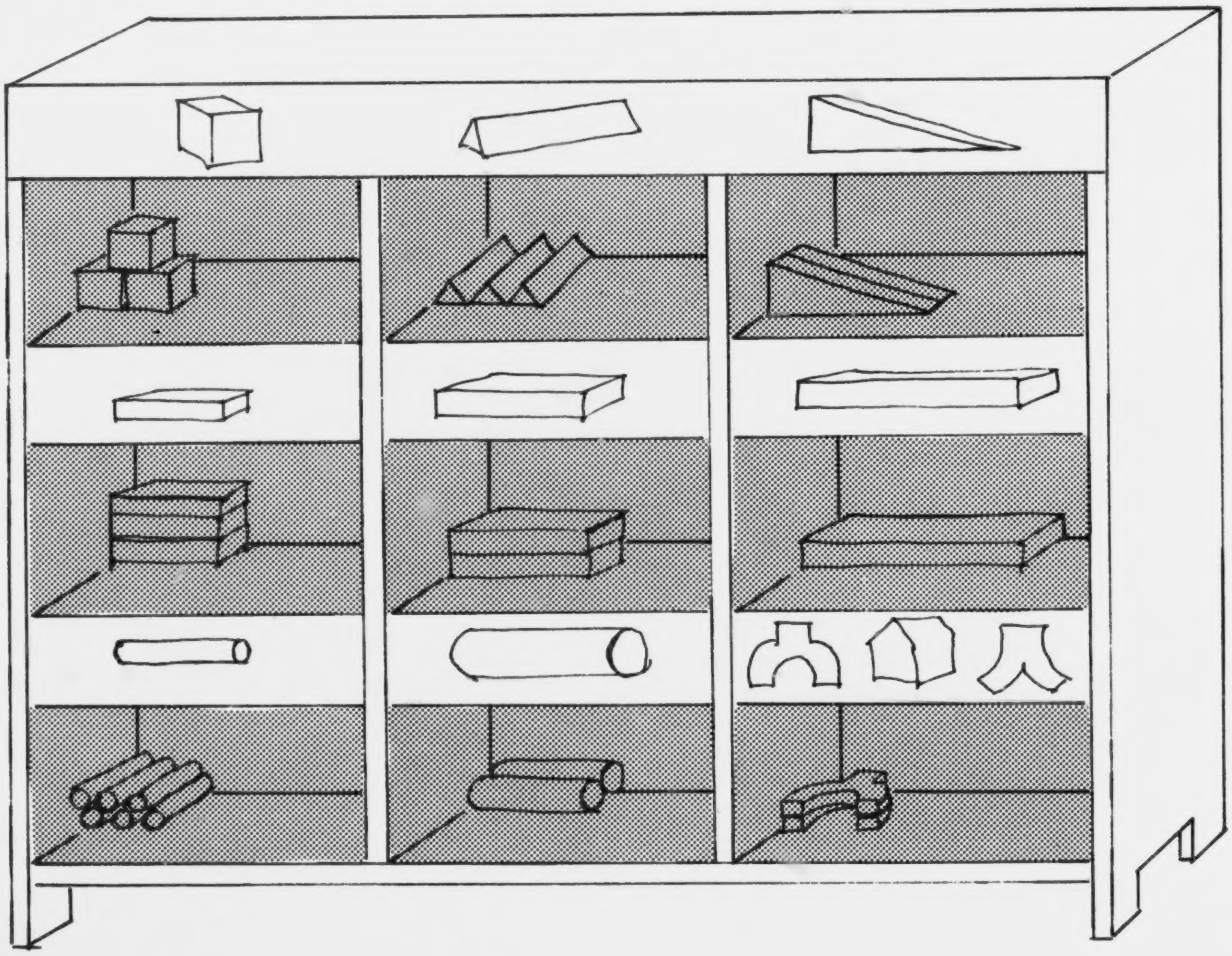
Exposure to
Classification of
Objects

Objective

Given the appropriate storage facilities, the child is able to return objects to their designated places.

ACTIVITIES

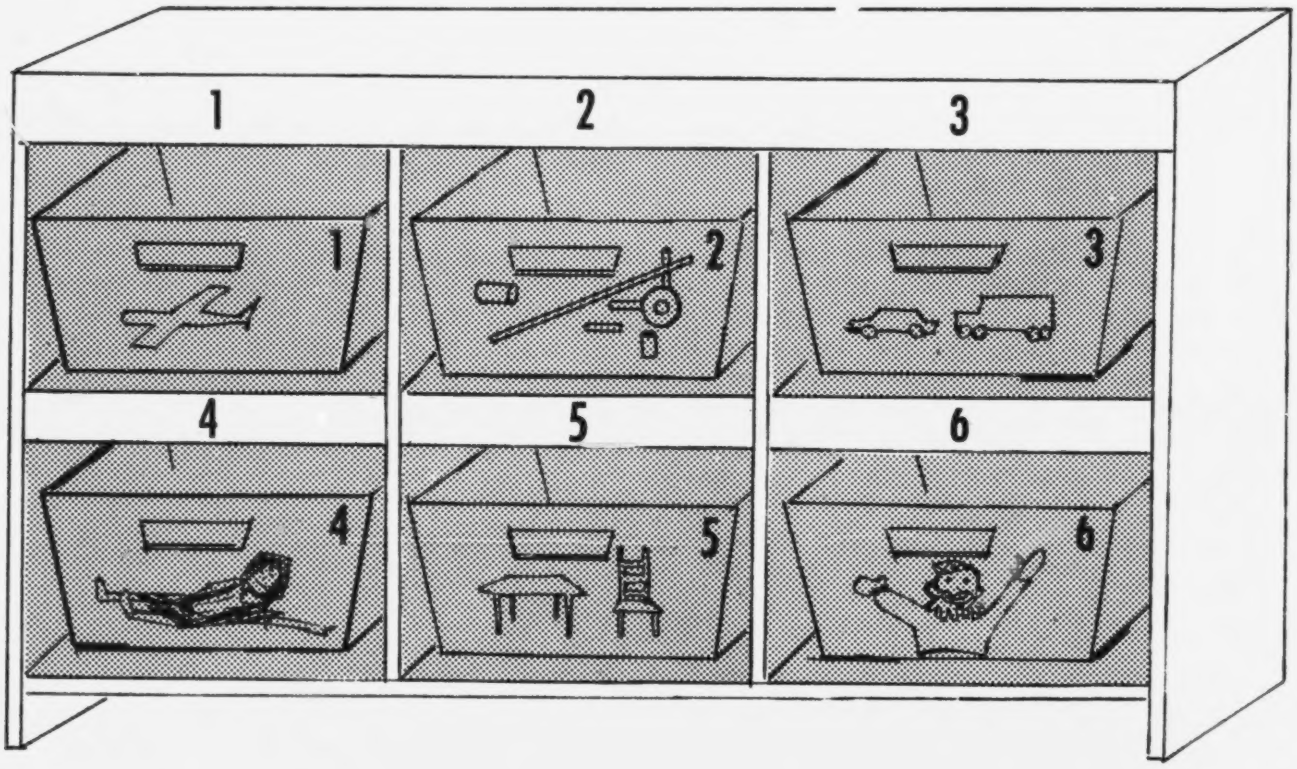
1. The space in which children are to work, play, experiment, listen, and rest should be organized in a manner which promotes the sorting of materials. One of the first tasks of the beginner is to learn that items should be kept in their designated places. This is a responsibility of every member of the group. For the storage of blocks, for example, provide picture labels above each storage area as pictured below to show the children where to put blocks of different shapes and sizes.



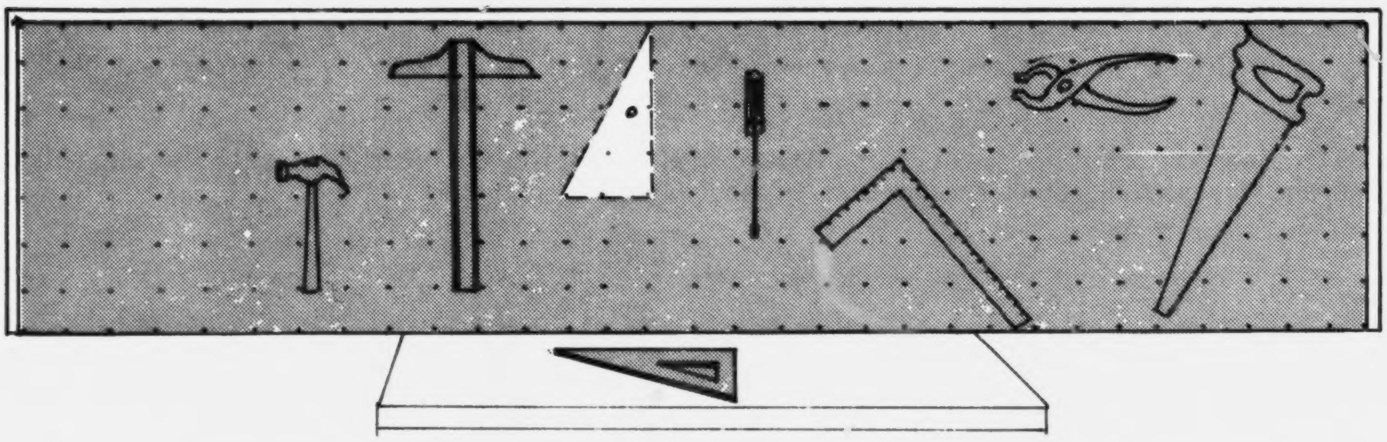
2. Art materials are frequently stored in flat bins or boxes within easy reach of children. A picture on the end of the container helps the child to know where to put scissors, paste, crayons, magazines, paper, clay and other items in this category.

(Continued on next page)

3. Containers which fit into designated shelf spaces will make clean-up time easier. A picture on one end of each container will expedite the sorting of toys. The containers sketched below are for toy planes, tinker toys, small cars and trucks, doll house families, doll house furniture, and puppets. Another learning experience has been included by writing a numeral on each container and a matching numeral above the space where that container is stored.



4. The big playhouse area offers additional opportunities for arranging and sorting objects. Dishes, silverware, dress-up clothes, and other items should be available in designated places.
5. The workshop with its tools, nails, pieces of wood, and measuring devices provides another place where children must sort and store materials in specific areas. Children can be directed to store tools on a pegboard which has the outlines of tools painted on it.



Content

Exploration and Discovery of Basic Ideas and Language of Sets

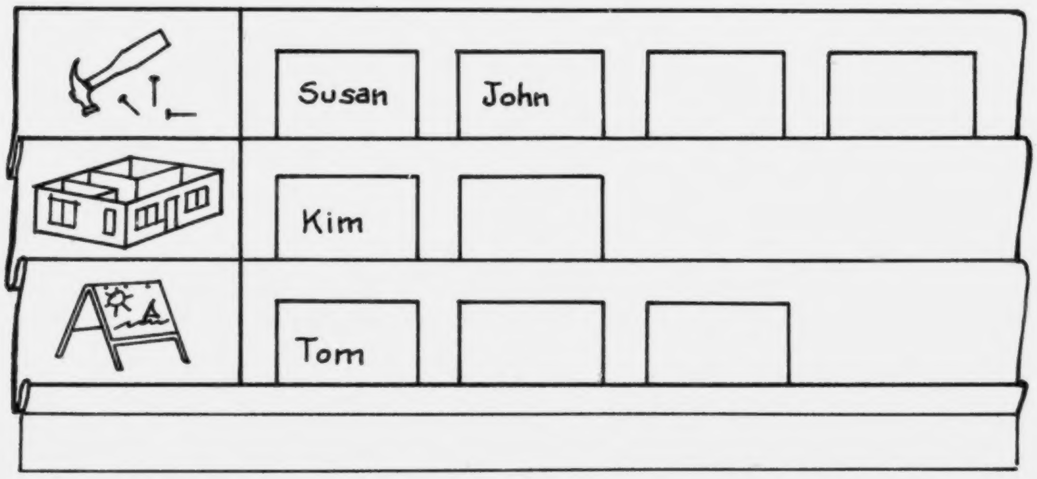
Objective

Given experiences in sorting and classifying things in his environment and hearing the appropriate language, the child can use this language to identify sets around him.

ACTIVITIES

1. In conversation with children as they play in the playhouse, in the workshop, or on the playground, ask: "Can you name the dishes in your set?" "Can you name the tools in this set?"

2. To help children learn that everyone has to take turns and make choices, limit the number who can work in the workshop, play with the dollhouse, listen to records, or participate in other activities. A cardholder such as the one illustrated below indicates the work and play areas in the room and the number of children each can accommodate at one time. As children select areas, place their name cards on the chart. Refer to them as "the set of children in the workshop," or "the set of children at the easel." Use the term "member of the set" when it is natural to do so, for example, "Jim, were you a member of the set of children in the playhouse?"



3. Have several children play a "Find a Set" game. Ask each child to select a set, tell what it is, and name its members. Begin with an example: "I have chosen a set of toys. The members of this set are a truck, a wrecker, and a doll. Dale, show us your set and tell us about it."

   A collection box provides interesting items from which the children may assemble sets. Encourage discussion about the members of the various sets.

Content

Meaning of a Set

Objective

Given a verbal description of a set, the child can distinguish between members of a set and things which are not members.

ACTIVITIES

1. Involve a few children at a time in making sets. Use small items which may be handled easily. Ask the children to select a set of

    a. things that are hard

    b. things that roll

    c. things that are soft

    d. things that make noise

    Give each child a piece of colored yarn and ask him to circle his set with it. Let the children tell about their sets, classify them, and identify the members.

    Now ask the children if other things could be members of the set. Would a pony be a member of a set of things that roll? Would a baseball be a member of a set of things that are soft? Let the children name other things which would or would not be members of the different sets they have made.

2. Provide opportunities for children to make decisions about sets. Can the same item be a member of more than one set? Who are the members of the set of all the children in this room? Of all the boys in this room? Of all the teachers in this room?

3. Let the children use a flannel or magnetic board to arrange sets which they have named. If several sets are shown at the same time, use pieces of yarn or magnetic ribbon to separate them.

Content

Matching
One-to-one
Correspondence

Objective

Given two equivalent sets of objects or pictures, the student can demonstrate a one-to-one matching between members of the sets by physically associating the objects or pictures.

ACTIVITIES

1. Show the children six plastic spoons and six empty baby food jars. Tell them that you want someone to show whether there are just as many members in the set of spoons as there are in the set of baby food jars. After a child has done the matching, have the children discuss the fact that for each jar there is a spoon and for each spoon there is a jar.

2. Use classroom experiences as they naturally occur to illustrate one-to-one correspondence. Some examples are the distribution of milk, chairs, books, or paper to the children. These are but a few of the many opportunities occurring daily which illustrate this concept.

3. Ask the children to show that they have just as many fingers on one hand as on the other.

4. Name two sets of children with the same number of members in each set. Direct one set to form a circle. Have the members of the other set stand inside the circle. Give each child in the outside circle a piece of string. Have a member inside the circle take the other end of the string. Continue to do this until all members of the outside set have been matched with the members of the inside set. Now ask the children, "Are there just as many members in one set as the other?" Have them explain their answer.

Content

Matching
Comparison of Sets

Objective

Given two sets of different size, the child is able to identify the set which has more members and the set which has fewer members.

ACTIVITIES

1. Select pairs of sets such as

   a. girls with ribbons in their hair, boys with a broken arm

   b. boys wearing long pants, boys wearing short pants

   c. the grown people in the room, the children in the room

   Use these sets to give the children practice in making decisions about whether one set has more members than another, fewer members, or the same number of members.

2. Have children work in pairs. Give them sets whose members are related but make them sets which cannot be matched one-to-one. The objects in these sets may be straws and cartons of milk, cups and saucers, or dolls and doll hats. Ask the children to match their sets. They may be expected to use such terms as "more than," "not as many," and "bigger than" in describing the differences they discover.

3. Play a guessing game with a small group of children. Use a random arrangement of sets on the floor, table, or flannel board. Circle each set with a piece of colored yarn. Have some sets with the same number of members, other sets with different numbers of members. Ask a child to choose two sets and identify them. Then ask him to guess whether they match. After he answers ask him to prove his answer. Let him try it his way. Have pieces of yarn available in case he wants to use it to show one-to-one matching.

4. Have children collect pictures or toys belonging to the following pairs of sets.

   (a) animals that live at the zoo, animals that live on the farm
   (b) toys with wheels, toys without wheels
   (c) things that magnets pick up, things that magnets do not pick up
   (d) things that float, things that do not float

   Match the members of these pairs of sets and discuss the matching. Children should be able to tell whether there are more members in the set of toys with wheels than in the set of toys without wheels, or whether there are fewer members in the set of things that float than there are in the set of things that do not float.

SETS A-6

Content

Ordering of Sets
The "one-more" Pattern

Objective

Given sets of different sizes, the child can arrange the sets in order.

ACTIVITIES

1. Arrange on a table or the floor 5 non-equivalent sets of objects. These might be blocks, pencils, crayons, pieces of chalk, stones, beads, bottle caps, beans, blocks, discs, or paper clips. None of these sets can be paired exactly with another, but let the children match different pairs of sets to discover this for themselves. If the children are having difficulty doing this you might suggest putting the members of one set in a row and another set in a row just below the first.

   When the children have had time to match the sets ask, "Could you match any two sets exactly?" Help children to see that each set has more or fewer members than another set.

   Encourage children to find the set with the fewest members and place it first, followed by the set having just one more member. Continue this until the sets have been arranged in order of size.

SETS A-7

Content

Empty Set

Objective

Given an example of a set which has no members, the child can describe it verbally.

ACTIVITIES

1. Ask the children to name the people or things in the room belonging to these sets:

   (a) boys with red hair
   (b) toys with wheels
   (c) teachers
   (d) things to cook in
   (e) pans with cake in them
   (f) girls with blue shoes

   Some of these sets will have no members. Ask the children if they can think of a good name for such a set. Perhaps they will call it *the empty set* before you introduce the phrase.

2. Continue this activity by asking children to name the members of the following sets: their brothers, their sisters, their pets, their teachers. Whenever a child has no members in one of these sets, have him describe that set as *the empty set*.

3. Children will enjoy thinking of other examples of the empty set. They will probably give you more interesting answers than these:

   The set of giants in the room.
   The set of snowballs on the windowsill.

Content

Meaning of a Set

Objective

Given a verbal description of a set, the child can distinguish between members of the set and things which are not members.

ACTIVITIES

1. Provide the children with old magazines, catalogs, paper, paste, and scissors. Direct each child to cut out a set of pictures to represent the members of his family including himself. Have the children make a family set by pasting these cutouts on a sheet of newsprint. Since each child is accustomed to think of his family as a group and since the family is such a personal relationship it will be easy for a child to distinguish between members of his set and those who are not members.

2. Paper plates and a variety of objects can be used to help children form sets. Give each child a paper plate and a number of objects to place on his desk, for example, crayons, scissors, pencils, coins, magazine cutouts, or paper discs. Have the children place a set of objects in the paper plate and then describe the set shown, for example:

   "This is a set of crayons."

   "This is a set of pencils."

   "This is a set of pennies."

3. Have children bring in sets of objects such as rocks, shells, coins, books, marbles, jacks, plastic animals, ships, planes, autos, or dolls. Many opportunities will occur to discuss members of the sets as children examine each other's collections.

4. Have the children help you make a collection of different pictures of animals, birds, people, wagons, cars, and airplanes. Give one to each child and when you give a description of a set to which his picture belongs have him stand showing his picture. Use descriptions such as

   "The members of the set I am thinking of can fly."

   "Members of my set can walk on two legs."

Content

Matching

One-to-one Correspondence

Objective

Given two equivalent sets consisting of pictures or objects, the child can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.

ACTIVITIES

1. Place four chairs at the front of the room. Ask a child to choose a set of children that has just as many members as the set of chairs and to assign a child to each chair. Have the class observe that there is a chair for each child and a child for each chair.

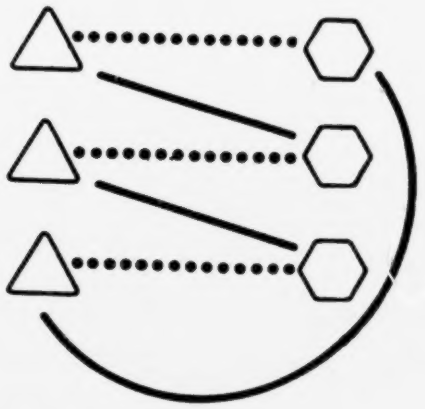
   Continue the activity with other pairs of sets and have the children actually match the members of these sets. For example:

   - A set of boys and a set of girls.
   - A set of papers and a set of pencils.
   - A set of girls and a set of books.
   - A set of boys and a set of toys.

2. Illustrate various sets on large sheets of tagboard. Show one of these to the children and then have them place a set of objects on their desks that has just as many members as the set you have just shown.

   Small cards of tagboard illustrated with sets of objects may be given to each child. With objects from his desk and string he can show a one-to-one correspondence.

3. Provide each child with a large sheet of newsprint and crayons. Direct the children to draw pairs of sets containing the same number of members. The teacher may wish to dictate the number for each pair. Ask the children to show a one-to-one matching between the members of the sets by drawing lines. (You may wish to encourage them to show more than one way of matching by using different colored lines as indicated by the dotted and solid lines below.)



Content

Matching
Comparison of Sets

Objectives

Given two sets of objects or pictures, the child can

1. identify them as having the same number of members or not, and
2. tell which set has more and which has fewer members when the sets are of different sizes.

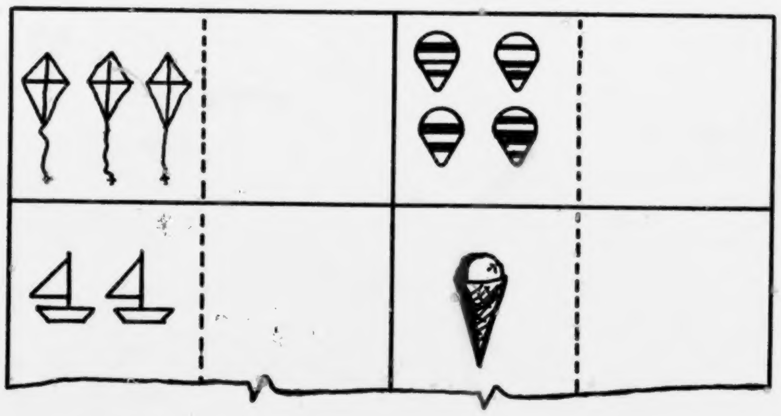
ACTIVITIES

1. An egg carton or a similar box with a lid and dividers is an ideal container in which each child can keep small objects such as checkers, plastic animals, buttons, stones, or cubes for set demonstrations.

   Place a set of three objects on the flannel board or magnetic board and ask each child to make a set containing the same number of members on the lid of his container or on his desk. After several equivalent set demonstrations, have the children make sets containing more or fewer members than the one shown on the flannel board.

   The children may then work in pairs making sets. They can compare them and decide which sets have more, fewer, or the same number of members.

2. Provide each child with a work sheet similar to the one below. Have them use their boxes of small objects described in the activity above. Next to each picture of a set is an open area. Have the children use this region to form a set which has more objects than the set in the picture. This may be done using objects from his container, by pasting pictures cut from old magazines, or by making drawings.



Content

Ordering of Sets
The "one-more" Pattern

Objective

Given ten sets of different sizes, the children can arrange the sets in order.

ACTIVITIES

1. Give each child 10 pieces of cardboard or art paper about 6" x 9". Have each draw a picture of a set for each number from one to ten. You may ask the children to shuffle their cards and then arrange them on their desks in order. These cards may be stored in individual envelopes for later use. Number words and Arabic numerals may be written on the backs of the cards.

2. Distribute ten cards such as those prepared in Activity 1 to groups of ten children giving one card to each child. Assign each group a section of the room and direct them to arrange themselves in numerical order.

3. Prepare illustrations of ten sets of different sizes on strips of tagboard. (Several sets of these should be made.) Place these randomly in a pocket chart and have the children arrange them in order. If a pocket chart is not available the strips may be placed on a table, desk, or floor.

4. Have the children work in groups using several small boxes. Ask the children to arrange the boxes in a row and to place discs, beads, or dried beans in them in such a way that no two boxes will contain the same number of objects. Then have them place these sets in order beginning with the set that has the fewest members, the empty set. For students working with eleven boxes to form sets with from zero through ten members, there must be at least 55 discs or beads available to use in making these sets.

SETS B-5

Content

Empty Set

Objective

Given a set such as all the children in the room who are grandfathers, the child can describe it as a set with no members.

ACTIVITIES

1. Use questions like these to challenge children to describe the empty set:

    a. What is the number of purple elephants in the classroom?

    b. How many days of the week begin with "A"?

    c. How many students in our room are over 10 years old?

    Encourage the children to give other examples to describe the empty set.

2. Using three boxes and two pencils you can show an empty set or let a child demonstrate an empty set. Place one pencil in each of two boxes and ask the class if any of the three boxes represents the empty set. Let each child demonstrate the empty set on his desk using his crayon box and crayons.

SETS B-6

Content

Parts of a Set
Subsets

Objective

Given a set such as all the children in the classroom, the child can identify a part of the set, for example, all the girls with red hair.

ACTIVITIES

1. Ask all the children in the classroom to think of themselves as a set. Describe that set and name people or objects that are not members of it. Then consider how this set can be subdivided into other sets such as children with brown hair, children wearing tennis shoes, etc. Continue this discussion with such questions as, "Can you call the set of children in the first row a part of the set of children in our classroom?" (Yes) "Why?" (Because every child in the first row also belongs to the set of children in the classroom.)

2. Have children cut out pictures of people, vehicles, animals, and fruit from magazines and paste them on tagboard. By circling portions of a picture with pieces of yarn they can show parts of the given set.

3. Use the calendar as another method of exploring parts of a set. The days of the week are a set; the school days, a part of that set; and the weekend, still another part.

Content

Meaning of a Set

Objective

Given a verbal description of a set, the child can distinguish between members of the set and things which are not members.

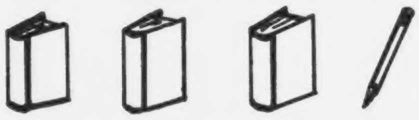
ACTIVITIES

1. Select a set of four or five girls and boys. Write their names on the chalk board. For example, you may write: Sue, Judy, Joe, John, and Don. They belong to the set and, therefore, are members of the set. Be sure to make it clear that the set consists of Sue, Judy, Joe, John and Don simply because they happened to be selected as the members of this set. Ask children to describe the set. Then have them name people or objects that do not belong to the set.

   Follow this activity by arranging the children in groups of four or five students. Have each group think of a set and then list the members of that set on the chalk board. Ask each group to describe its set in various ways and to name people or objects that are not members of the set.

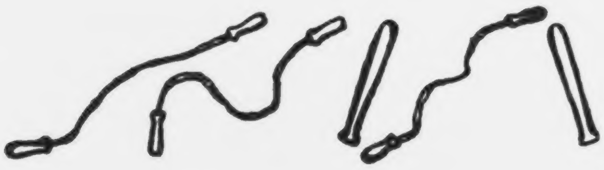
2. Display sets of objects with one or more elements that do not belong in the sets as labelled. Have the children remove those objects which are not members of the set named.

   a. Books:



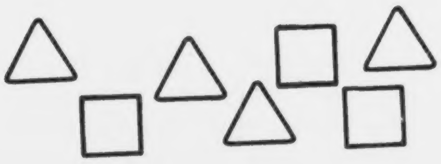
(Remove the pencil.)

   b. Jump ropes:



(Remove the bats.)

   c. Triangular shapes:



(Remove the squares.)

3. The children will enjoy bringing in objects they have collected. This can easily develop into an amusing way of discussing sets.

Content

Matching
One-to-one
Correspondence

Objective

Given two equivalent sets of pictures or objects, the child can demonstrate a one-to-one matching between members of the sets by constructing lines or by physically associating the objects.

ACTIVITIES

1. Provide the following materials for each child:

   5 discs or cutouts
   5 pieces of string
   5 cards with the numerals 1 through 5 written on them
   a box of crayons or other objects

   As a child or the teacher is demonstrating this activity, the rest of the class can participate at their desks.

   Form two sets on the flannel or magnetic board. The first set will contain 5 objects and the second set will contain 5 different objects, for example:

   Set A: horse, kite, train, pear, ball

   Set B: bird, dog, cat, horn, girl

   Use string to show that it is possible to match each member in Set A with one and only one member in Set B and every member in Set B with exactly one member in Set A. Therefore, Sets A and B are in one-to-one correspondence with each other.

   Next replace the second set with the first five counting numbers and call it Set C

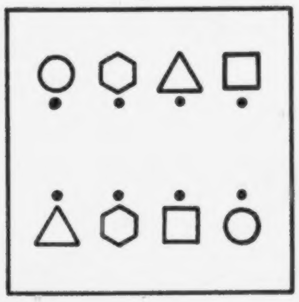
   Set A: horse, kite, train, pear, ball

   Set C: 1, 2, 3, 4, 5

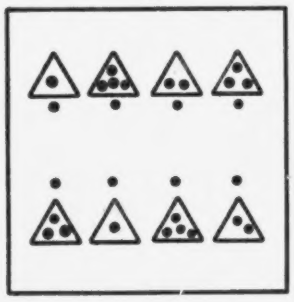
   Many other activities using concrete objects in this manner will help reinforce this concept of one-to-one matching.

2. Below are some examples of the kinds of activities you may give your students for additional practice in matching. Allow them to make their own discoveries by just getting them started on a and b below.

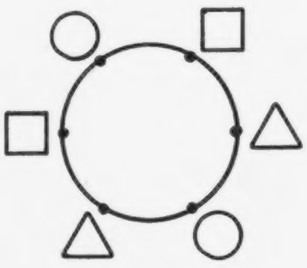
a



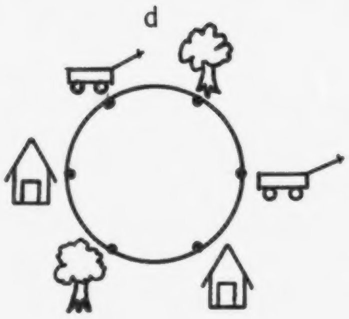
b



c



d



A similar activity is illustrated by c and d where the members of the sets to be matched are not in rows.

Content

Matching
One-to-many
Correspondence

Objective

Given several familiar illustrations involving a one-to-many matching, the child can describe where the one-to-many matching occurs, for example, one bicycle to two wheels.

ACTIVITIES

1. Describe such a matching as one bicycle to two wheels. Challenge the children to see how many different examples of one-to-many matching they can find. Some examples are:

   a. One chair to four chair legs.

   b. One foot to twelve inches.

   c. One roller skate to four wheels.

   d. One table to four legs.

   e. One face to two eyes.

   f. One foot to five toes.

   g. One teacher to twenty-five students.

Content

Matching
  Comparison of Sets

Objectives

Given two sets of objects or pictures, the child can

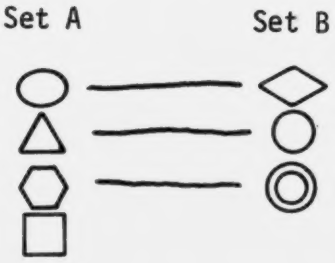
1. compare the number of members in each set, and
2. tell which has more and which has fewer members when the sets are not the same size.

ACTIVITIES

1. Divide the class into two sets, one of boys and the other of girls. The children will enjoy discussing whether the sets are equivalent or non-equivalent. Which set has more members? . . . fewer members?

   After doing the activities just described or similar ones, children who are having no difficulty are ready to move into more abstract activities such as marking or drawing equivalent sets.

2. Using a flannel board, magnetic board, discs on the overhead projector, or any concrete objects on your desk, show two sets. Name them Set A and Set B. Ask a child to pair these sets by using string or by drawing lines.



   The children may use objects at their desks to make sets equivalent to these.

   The class will see that one member of Set A is not used in this pairing. Give the children an opportunity to make statements as they compare the two sets. In this example they may say that Set A has more members than Set B or that Set B has fewer members than Set A.

3. Use a box of dominoes to illustrate equivalent and non-equivalent sets. The dots on a domino may be considered a set.

   a. Have the children select dominoes at random two at a time and place them on large sheets of paper. When the dominoes do not represent equivalent sets of dots, instruct the children to draw a circle around the larger set.

   b. Have a child divide a sheet of paper into two parts. Place pairs of dominoes illustrating equivalent sets on one part, and pairs of dominoes representing non-equivalent sets on the other part.

Content

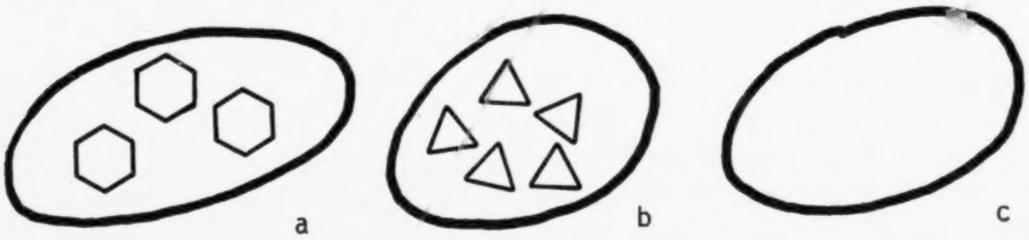
Empty Set

Objective

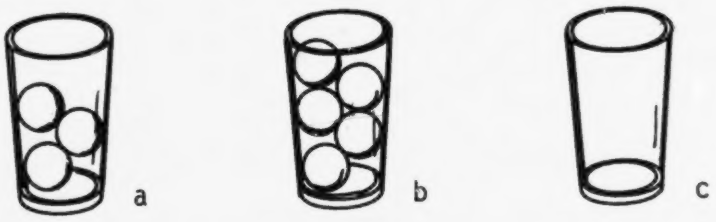
Given an example of the empty set, the child can describe it as a set which has no members.

ACTIVITIES

1. Present several examples of sets to your children.



Ask them how many objects are in each set. Example c above should bring out the word "empty" from some of the children. This answer is more likely to be given if transparent containers are used as shown below.



Set Containers

Marbles or ping-pong balls can be used as members of these sets. The glass which contains no objects is then a good example of the empty set.

2. Have your students name other examples of the empty set. You may get them started by asking them to consider the set of all children in their class who are as tall as a giraffe. Since none of them will be this height, this is a good example of the empty set.

Content

Parts of a Set
Subsets

Objective

Given a set, the child can identify parts of the set.

ACTIVITIES

1. Have children place a set of objects on their desks. You may supply a set of different colored discs or may ask children to make their own sets. Have each child describe his set to his partner and show him parts it can be divided into. The empty set can be considered since it is a subset of every set. As children are exploring their sets with their partners, you can work with those having difficulty.

2. Arrange several baskets on a table. Place objects in some of the baskets; have others empty. Those baskets containing objects form one subset, and the empty baskets form another subset.

Content

Meaning of a Set

Objective

Given a description of a set, the child can distinguish between things that are members of the set and things that are not members.

ACTIVITIES

1. Review with your students the following words: groups, collections, sets. Adapt the concept of sets to ordinary home situations. In the home there are many groups or sets of objects. Children use the word *set* frequently. Have them list some of the more common uses of this word such as:

   a. The set of dishes in the kitchen

   b. The set of woodworking tools in the workshop

   c. The set of toys in a toy chest

   d. The set of Christmas decorations

   Suggest to your children that they name sets of objects found in the classroom. Then have them name particular objects or members that make up the sets they have listed. This may be done as shown below.

Name of Set	Possible Members (answers will vary)
a. The kitchen dishes	saucers, dinner plates, soup bowls, etc.
b. Woodworking tools	hammer, saw, plane, chisel, etc.
c. Your toys	ball, stuffed animal, skates, etc.
d. Christmas decorations	lights, tinsel, ornaments, wreaths, etc.

2. Have the students underline the elements which do not belong to the given set:

   a. Children in the room: boys in the room, girls in the room, teacher

   b. Insects: butterflies, spiders, ants

   c. Fish in the aquarium: goldfish, snails, guppies

   d. Vegetables in a salad: oranges and apples, lettuce , carrots, celery, cabbage.

Content

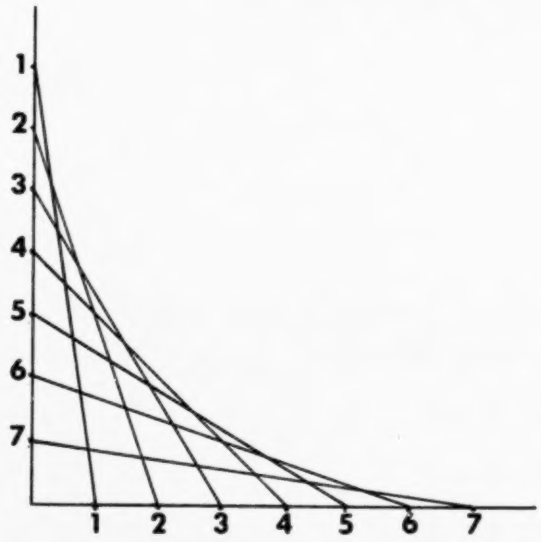
Matching
One-to-one
Correspondence

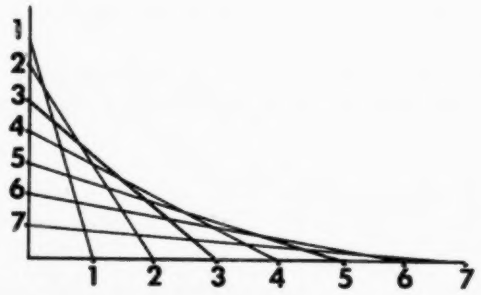
Objective

Given equivalent sets either as actual objects or picture or word descriptions, the student can demonstrate a one-to-one matching between members of the sets either by drawing lines or by physically associating the objects.

ACTIVITIES

1. Draw two segments which have a common endpoint and are perpendicular to each other, and label them as shown below. Use crayons to connect the corresponding points on the two segments. The resulting segments will give the illusion of curves.





*This idea can be used with brightly colored thread and tagboard. Pencil marks on the "back" can be used to provide a guide for the "needle". The resulting effect might encourage some students to create more complicated designs.

Content

Matching
One-to-many
Correspondence

Objective

Given several familiar physical situations involving a one-to-many matching, the child can describe where the one-to-many matching occurs, for example, one hand to five fingers.

ACTIVITIES

1. See if your children can give examples of one-to-many correspondence that exist in the classroom. Some possible observations are:

   a. One classroom to many children.

   b. One pencil sharpener to many holes for various sizes of pencils.

   c. One clock to twelve numerals.

   d. One towel dispenser to many towels.

   e. One closet to many shelves.

   f. One filing cabinet to several drawers.

2. Have the students select several numbers to name in different ways. By drawing arrows to these various names they will develop a one-to-many matching.

$$\begin{tikzcd} & 3 + 0 \\ 3 \arrow[ur] \arrow[r] \arrow[dr] & 2 + 1 \\ & 4 - 1 \end{tikzcd}$$

$$\begin{tikzcd} & 3 + 2 \\ 5 \arrow[ur] \arrow[r] \arrow[dr] & 5 \times 1 \\ & 10 \div 2 \end{tikzcd}$$

Content

Matching
Comparison of Sets

Objectives

Given two sets of objects -- real, pictured, or described by words --

1. compare them in size, and
2. tell which set has more and which has fewer members if they are not the same size.

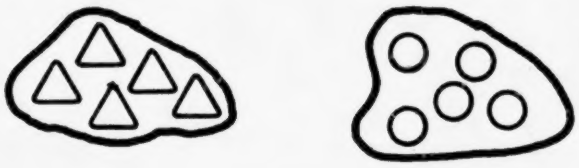
ACTIVITIES

1. Distribute to the children such objects as crayons, discs, or ice cream sticks. Have them form examples of equivalent and non-equivalent sets. By actually matching the elements of the sets, children can learn to classify them as equivalent or non-equivalent. This pairing of members of sets will enable children to see how two sets compare in size.

2. Distribute large pieces of paper to your children. Have them draw pictures of several pairs of sets with some equivalent and others non-equivalent. Use these pictures in class discussion to have children determine whether the sets shown are equivalent or non-equivalent.

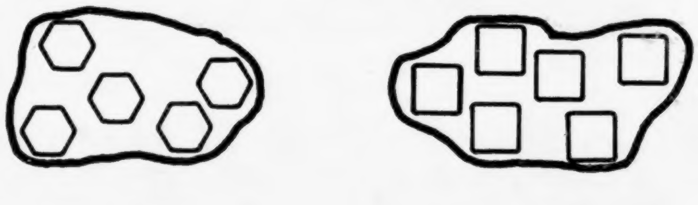
3. Provide your students with pictures of sets such as those shown below. Have them indicate whether the pairs of sets are equivalent or non-equivalent.

a.



(Equivalent)

b.



(Non-equivalent)

c. {1, 3, 5, 7, 9} {2, 4, 6, 8, 10} (Equivalent)

d. {Tom, Jack, John} {Pat, Mary, Sue, Gail} (Non-equivalent)

Content

Empty Set

Objective

The student can describe verbally a set which has no members.

ACTIVITIES

1. Have each child place his set of crayons on his desk. Ask him to describe the set of silver crayons or any other color he does not have. You might do the same with books, pencils, or any other items which can be described that are not available.

2. Prepare a bulletin board or a flannel board displaying a set of objects such as animals, toys, community helpers, or any objects currently being studied in the classroom. Discuss why this display can be called a set. Ask the children to describe a set of objects not present. Their response might be, "We can't because there aren't any." Children can then give examples of the empty set. Zero as the number of the empty set should be included in the discussion.

Content

Parts of a Set
Subsets

Objective

Given a set, the child can identify parts of the set.

ACTIVITIES

1. Use materials in the classroom to illustrate a set and its subsets. After introducing this topic by using objects in the room, have each child make a collection of pictures showing various means of transportation. As you display a set of these on the flannel board, have the children work with their individual sets at their desks. Ask them to examine these individual sets to find subsets and then circle them with pieces of yarn. Some examples of subsets are:

   a. Members of the set which are used for land transportation.

   b. Members of the set which are used for water transportation.

   Since the pictures in a student's set will be different from those in other students' sets, there will be many opportunities for discussion. The two subsets described above are only two of many which exist. Your children will probably be able to name others.

SETS E-1

Content

Empty Set

Objective

Given the term, *empty set*, the student can name sets which have no members.

ACTIVITIES

1. Students may think of other examples themselves after seeing and discussing some of the following:

   a. The set of living men over 150 years old.

   b. The set of triangles with four sides.

   c. The set of purple cows in the classroom right now.

   d. The set of odd numbers contained in {2, 4, 8, 16}.

SETS E-2

Content

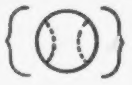
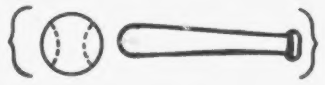
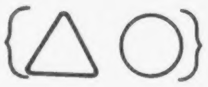
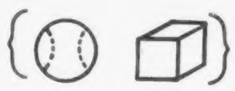
Parts of a Set
Subsets

Objective

Given a set, the student can identify subsets of the given set.

ACTIVITIES

1. Set A, B, and C below are each followed by three sets. Have your students underline those sets which contain the first set as a subset.

A =				
B =	{1, 2, 3}	{1, 3, 5, 7}	{4, 3, 2, 1}	{2, 4, 6, 8}
C =	{5, 10, 15}	{0, 5, 10, 20}	{10, 15, 20, 25}	{5, 7, 10, 12, 15}

2. Direct the students to give a subset for each of the following:

   a. The set of children in the room
   b. The set of books on a shelf
   c. {1, 2, 3, 4, 5}

Content

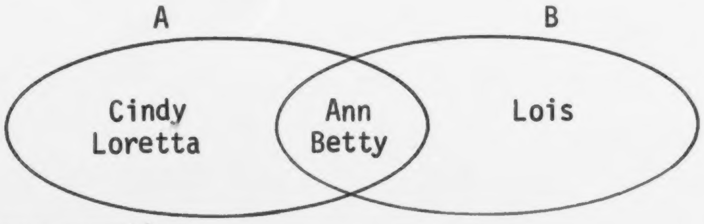
Combining Sets
  Union
Set Diagrams

Objective

Given two or more sets, the student can combine them to form a new set.

ACTIVITIES

1. The use of set diagrams is a helpful way to present the idea of the union of sets. For example: If A = {girls wearing white blouses}, and B = {girls wearing red skirts}, then the union of the two sets is all the girls wearing white blouses or red skirts or both.



Ask your students these questions:

a. According to this diagram, which girls were wearing white blouses?

b. Which girls were wearing red skirts?

c. Which girls were wearing white blouses or red skirts or both?

d. What is the union of A and B?

2. In the examples which follow ask your students to join the given sets and list the resulting set with its members enclosed in braces.

a. You will generate more interest in this activity by using the names of students in the class.

Given A = {Al, Bob} B = {Craig, Ed, Frank}

Answer: C = {Al, Bob, Craig, Ed, Frank}

b. Given D = {January, February, March} E = {April}

Answer: F = {January, February, March, April}

c. Given G = {1, 2, 4} H = {3, 5, 7}

Answer: I = {1, 2, 3, 4, 5, 7}

d. Given J = {1, 3, 5, 7} K = {1, 2, 3, 4}

Answer: L = {1, 2, 3, 4, 5, 7} (Notice that the 1 and 3 are not repeated in the union.)

SETS F-1

Content

Size of Sets
  Finite
  Infinite
  Empty

Objective

Given a set, the student can classify it as finite, infinite, or empty.

ACTIVITIES

1. Ask your students to classify the following sets by the number of elements contained in each. Have them explain their answers.

a. The set of all your classmates.	(finite)
b. The set of all even numbers.	(infinite)
c. The set of purple elephants in the classroom.	(empty)
d. The set of all odd natural numbers less than 30.	(finite)
e. The set of people in North Carolina who are over six feet tall.	(finite)
f. A = {1, 2, 3, 4, . . .}	(infinite)

SETS F-2

Content

Combining Sets
  Union of Sets

Objective

Given two or more sets, the student can form their union.

ACTIVITIES

1. Ask all students who have a dime to stand. Then ask those students wearing something green to stand. All students now standing represent the union of two sets. Several examples of this nature should help fix the notion of set union in the minds of your children.

2. After presenting the idea of set union display several sets on the chalk board and have students form the unions of these sets.

A = {Joe, Ann, Bill} B = {Bill, Bob, Mike} C = {Cathy, Joe, Ann, Jill}

a. $A \cup B$ = {Joe, Ann, Bill, Bob, Mike}

b. $A \cup C$ = {Joe, Ann, Bill, Cathy, Jill}

c. $B \cup C$ = {Bill, Bob, Mike, Cathy, Joe, Ann, Jill}

d. $A \cup B \cup C$ = {Joe, Ann, Bill, Bob, Mike, Cathy, Jill}

Content

Overlapping of Sets
Intersection

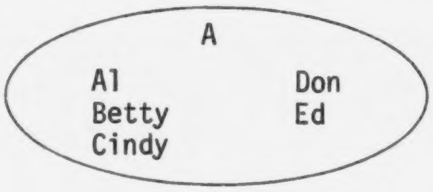
Objective

Given two or more sets which have common members, the student can name the intersection.

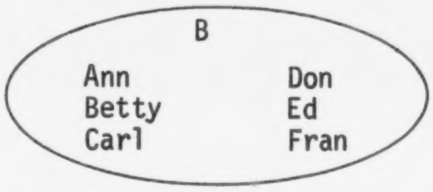
ACTIVITIES

1. Direct those students who like to play football to stand. Ask those who like to play basketball also to stand. Among the students now standing will be some who like to play both football and basketball. Ask the others to be seated. Those students left standing represent the overlapping of the two sets named. This activity repeated several times will explain the term *intersection*.

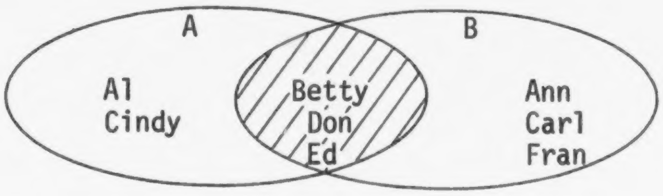
2. The use of diagrams depicting different sets and their overlapping is another way to introduce the intersection of sets. For example, list the members of your class who have two brothers. Call this Set A.



List the members of your class who have one sister. Call this Set B.

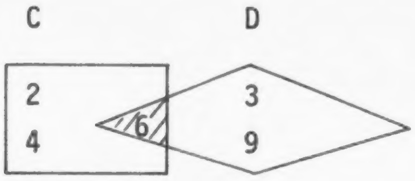


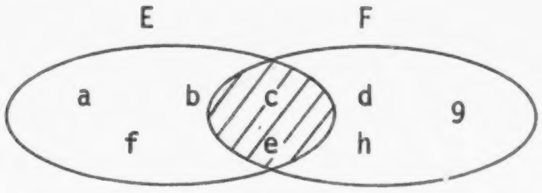
Are there any students who have two brothers and one sister? This can be shown as:



The shaded part shows the intersection of the sets.

*3. Have your students solve the following problems.





$C = \{2, 4, 6\}$
$D = \{3, 6, 9\}$
$C \cap D = \{6\}$

$E = \{a, b, c, f, e\}$
$F = \{c, d, e, g, h\}$
$E \cap F = \{c, e\}$

Content

Size of Sets
  Finite
  Infinite

Objective

Given examples of sets, the student can classify them as finite or infinite.

ACTIVITIES

1. The basic difference between a finite set and an infinite set is that the elements of a finite set can be counted with the counting coming to an end.

   If the elements of a set cannot be counted with the counting coming to an end then the set is said to be infinite.

   Note: At this level the abstract notion of infinity should only be approached intuitively.

   Ask the students to determine whether the following sets are infinite or finite.

   a. The students in your school. (finite)

   b. The earth's population. (finite)

   c. The natural numbers divisible by 5. (infinite)

   d. The number of bicycles in your city. (finite)

   e. The set of all fractional numbers less than 1. (infinite)

Content

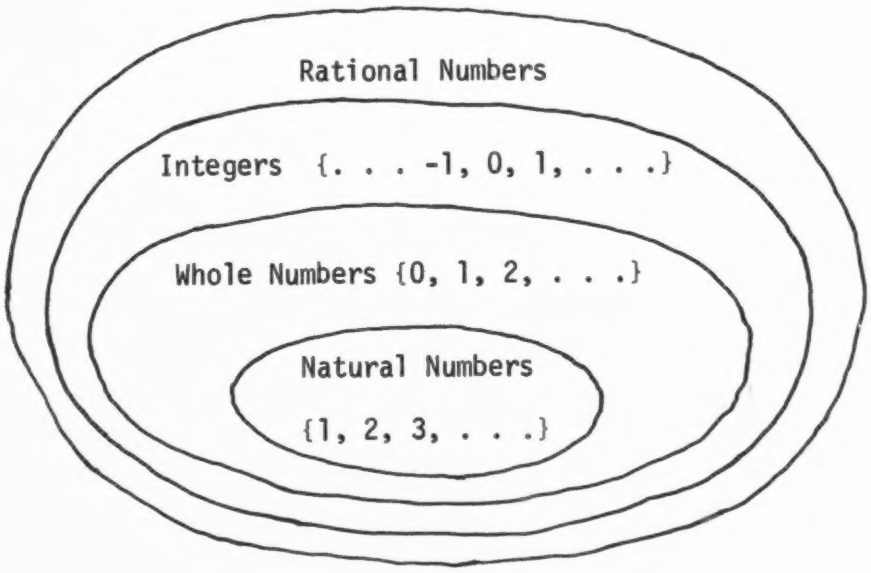
Set Diagrams
Subsets

Objective

Given several sets of numbers including the Natural Numbers, Whole Numbers, and Integers, the student can draw set diagrams to show how these sets relate to each other.

ACTIVITIES

1. This visualization of subsets is a good way to help students understand the relationships between these sets of numbers.


Rational Numbers
Integers {. . . -1, 0, 1, . . .}
Whole Numbers {0, 1, 2, . . .}
Natural Numbers
{1, 2, 3, . . .}

Numbers

Content

NATURAL NUMBERS

Exposure to
Cardinal and Ordinal
Use of 1 through 5

Objective

Given situations in which the child hears language which expresses the cardinal and ordinal use of numbers, he adopts it as part of his own vocabulary.

ACTIVITIES

It has been said that language is learned in the ear and secured on the tongue. We should therefore provide both exposure to the language of numbers and opportunities for using it early in a child's school life.

1. The teacher deliberately uses cardinal and ordinal number words in such natural situations as:
    a. Touching each child as she says, "Will these four boys - one, two, three, four - set the table, please."
    b. Asking the children to help her count the candles on a birthday cake.
    c. Urging the children to sort the blocks in groups of two, three, four, or five.
    d. Counting with the children as they jump ropes, bounce balls, hop, clap or do rhythmic activities.
    e. Using directions such as - "Let Tom walk the board first; Sue, you go second; Bill, third, . . . "
    f. Instructing children to put items away by giving them directions which designate order as, "Put the puzzles on the second shelf" or "Pick up the biggest blocks first."
    g. Encouraging children to verbalize their experiences, for example, "I walked first," "Bill was third," "My coat is on the second hanger."
    h. Asking the children to count with her the number of birds at the bird feeder.
2. Have the children line up in order, first, second, third, etc. Then have them turn around and ask, Who is first? . . . second? . . . and so on.
3. Each month on a large calendar, circle any special days such as children's birthdays or holidays. When these days are discussed suggest, "Let's count the days until Kim's birthday, . . . until Easter, . . . until Christmas," etc.
4. The teacher takes advantage of the children's love of repetition by providing opportunities for them to hear, sing, chant, and help tell:
    a. Familiar rhymes using numbers
    b. Well-known stories containing the vocabulary of numbers.

(Continued on next page)

c. Action songs involving counting such as the following:

Hickory-dickory-dock	(Swing arms like pendulum)
The mouse ran up the clock	(Raise arms overhead)
The clock struck one	(Clap hands 1 time)
The mouse ran down	(Drop arms to the side)
Hickory-dickory-dock	(Swing arms like pendulum)

Hickory-dickory-dock	(Swing arms like pendulum)
The mouse ran up the clock	(Raise arms overhead)
The clock struck two	(Clap twice)
The mouse said "Ker-choo"	(Sneeze exaggerated)
Hickory-dickory-dock	(Swing arms like pendulum)

Hickory-dickory-dock	(Swing arms like pendulum)
The mouse ran up the clock	(Raise arms overhead)
The clock struck three	(Clap three times)
The mouse said "Whee"	(Drop arms to the side)
Hickory-dickory-dock	(Swing arms like pendulum)

Continue, including a new rhyming word at the end of the fourth line for each new number.

Note: At this level children are being exposed to language which they imitate. They should not be pressured to do rote counting.

Content

NATURAL NUMBERS

Discovery and
Exploration in
Counting and
Comparing

Objectives

Given an environment containing a variety of activities, the child

1. hears,
2. responds, and
3. uses

language dealing with comparisons such as in the question, "how many?"

ACTIVITIES

1. Playing is the serious work of childhood.

   a. As you observe children playing with blocks, you may ask such questions as:

      How many more do you need?
      Which house do you think is bigger, yours or Tom's?
      Do you have three of this kind?
      How many blocks do you think we have here?

   b. While with the children in the playhouse, you may ask:

      May I join this group?
      Alice, how many children do you have?
      Barbara, do you have more children than Alice?
      Bill, how many plates will you put on the table?
      Joe, whose carriage has four babies in it?

   c. In the art center you may ask:

      Who has three clowns in a picture?
      Is there less blue paint than red paint?
      Do we need more paper for this group?

   d. As small groups dramatize a story you may ask them:

      Who wants to be a bear?
      Who wants to be an elephant?
      Who wants to be a horse?
      How many bears, elephants, and horses do we now have?

2. Children may guess how many other children they see on the playground or in the lunchroom, how many ants there are in the ant colony, or how many dandelions there are on the lawn.

Content

NATURAL NUMBERS

Counting
1 through 5

Objectives

Given a set of objects, the child can

1. count the members of the set and
2. say the corresponding number as he touches each member.

ACTIVITIES

1. The ability to say the number words in sequence and the ability to match the saying of the word with the touching of an object do not necessarily develop simultaneously. Thus you should provide many opportunities for children to count by touching and separating items as they count. Begin with big objects.

   Give directions such as:

   a. "Select some children to help you. Count them."

   b. "Count the children in this group; count the boys in this group; count the girls; count enough napkins for each child at this table; count enough straws and spoons."

   c. Play a game, "Give Me" with a small group. "Give me three crayons" - "Give me five small blocks" - "Give me four buttons." Have the children take turns naming items to be counted.

2. After they have had experiences counting larger things, provide a wire with five beads for them to move from one end to the other as they count. Partners may play "Show Me." One gives the command "Show me five beads;" the other counts them and then they exchange roles.

3. Children whose muscular coordination permits may join to make a group to do any of the following things a given number of times: bounce a ball, jump a rope, touch their toes, jump into and out of a circle or hoop. Give the children time to practice bouncing, jumping, and skipping before requiring counting along with each activity.

4. Children may pretend to buy and sell items such as lollipops, pieces of gum, or candy for which they pay pennies. They can count the items and the pennies.

5. In the playhouse area identify a set of children. Ask them to count the correct number of dishes, spoons, and napkins, until the table is ready for the tea party.

*Note: Children who are ready will extend their counting beyond five; those who are not will be happy with touching and counting no more than five in many different settings.

Content

NATURAL NUMBERS

Cardinal Use
1 through 5

Objectives

The child can

1. recognize numerals and
2. match them with the corresponding number of objects.

ACTIVITIES

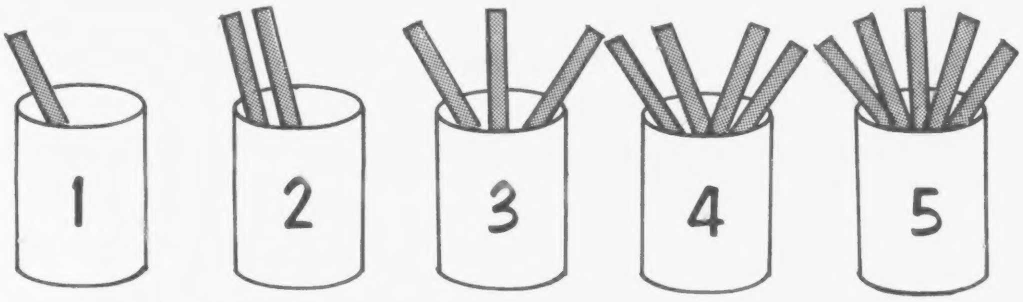
Many children are aware of numerals without understanding the numbers which the numerals represent. Therefore, no attempt should be made to have children at this level write number names or symbols.

1. Work with a small group of children so all may participate. Use a flannel or magnetic board and objects which adhere to it. Ask different children to select a set consisting of one duck, two stars, or three clowns and place it on the board.

   Now tell them that you are going to find the numeral that shows how many are in each set. Use large numerals and place them near the appropriate sets. Say the number as you show the numeral. Change roles, you arrange a set of objects and let the children take turns showing the matching numeral.

   Children who do this readily may include the numerals 4 and 5 in their game on succeeding days. Provide a variety of practice and use additional numerals only as a child indicates that he is ready for them.

2. As a variation of the activity above, provide a set of small containers with numerals written on them. Children may use tongue depressors, straws, or ice cream sticks to represent the numbers indicated by the numerals on the side of the containers.



3. Make up games that call for small groups of children to recognize numerals found in the room. For example: "I spy the numeral 5. Guess where?" (It may be on the calendar, the clock, the birthday chart, etc.)

4. Use a set of 5 cards, each about 8" x 8" with a large numeral on it: Ask a few children at a time to join you for a game called "Name the Set." Begin by saying "I see a set of paint brushes. Can you find a numeral card that names the number of that set?" Let a child select the card and put it on the tray with the brushes. Suggest that the children take turns finding sets of things in the room which they can match with one of the numerals. (Dolls in a carriage, children in the playhouse, children looking at books, and magazines.)

5. Use flash cards with numerals on them. Ask children to do something the same number of times that the numeral shows. Start by suggesting tapping, hopping, etc. Then let children be creative and name their own activity.

Content

NATURAL NUMBERS

Cardinal Use
1 through 5

Objectives

Given a set with as many as 5 members, the child can

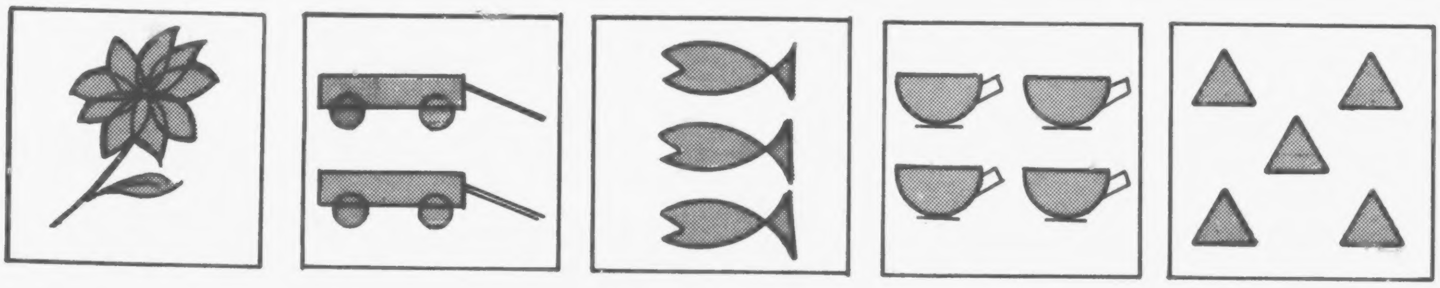
1. name the number of objects in the set and
2. select the correct numeral.

ACTIVITIES

Competitive games are not recommended for all young children and, therefore, should be used with caution.

1. Invite a small group of children to use the flannel or magnetic board to show some sets and name them. Sit on the floor in an area where no other activity will interfere. Suggest that one child begin by showing a set of not more than five objects. He may choose someone to name the number of objects and select the numeral that shows how many.

2. Prepare a set of 8" x 8" picture cards to match the numeral cards from 1 through 5.



Explain the two sets of cards by calling one the "set of numeral cards," the other the "set of picture cards." Shuffle each set; give the numerals to one player and the pictures to another. Show them how to play a matching game in which the first player places one of his cards face up and his partner matches this with a corresponding card, naming the number of objects verbally as he does this, as "This is a picture of a set of four cups."

When such a game as this has been introduced the teacher should supervise. Later she should give these children the opportunity to explain it to other children.

3. As a variation of No. 2 the teacher may play with a small group of children. Give each child a set of smaller numeral cards (5" x 5" with numerals from 1 through 5). Show one of the picture cards or say (without cards), "I am thinking of a set of four girls. Show me the numeral four." Each child then selects the numeral (4) and holds it up. Then the teacher will show the correct large numeral card so that each child can compare his own selection with it.

Content

NATURAL NUMBERS

Cardinal Use
1 through 5

Objectives

Given a number such as 5, either spoken or written the child can
1. identify and
2. form

sets containing the given number of members.

ACTIVITIES

1. Select a small group of children to help you find some sets with from one to five members. Begin by saying, "I see a set of three girls with bands on their hair. Name these girls." Now let various members of the group identify other sets of 5 or less.

2. Have children "fish" for a numeral and then find a set containing that many members. Cut the fish from paper, write a numeral from 1 through 5 on each and attach a paper clip. Use a stick, a piece of string, and a magnet for a fishing pole.

Content

NATURAL NUMBERS

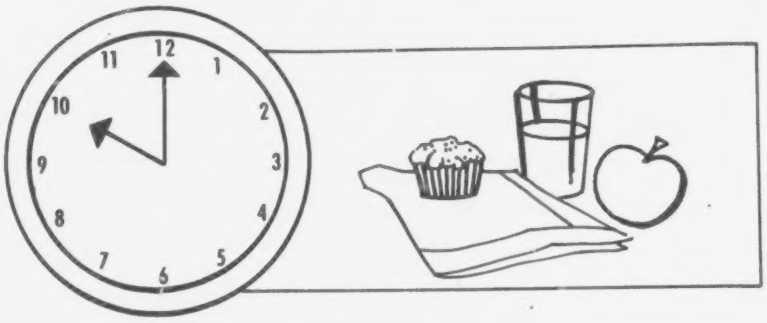
Objective

The child recognizes some of the uses of numerals which he sees around him.

ACTIVITIES

1. Before children are able to read a clock, a calendar, a thermometer, they should be aware of their functions.

   a. Draw a large clock face with the hands set for "snack time." Place an appropriate picture beside it. Ask children to watch the real clock so they can tell when it is "snack time."



(Continued on next page)

b. Record the birthdays of your children on a large calendar. On each birthday, call attention to the date and to the person whose birthday it is. Help children locate other special days on the calendar but do not expect all of them to be able to read them.

c. As you engage children in conversation about themselves, encourage them to answer the question, "Where do you live?" in this way, "I live at 247 Green Street" or "I live on Route 4, Cary." Record the addresses as you repeat them so children will begin to realize that numerals are used in this way. The same procedure may be used with telephone numbers.

d. Suggest that children look for places where numerals are seen in the classroom and other areas of the school. Discuss their findings and the uses of numerals. Some possibilities are: measuring devices such as a yardstick or thermometer, room designations, shuffleboard courts, and prices in the lunchroom.

NUMBERS A-8

Content

NATURAL NUMBERS

Order
1 through 5

Objective

Given a shuffled set of numeral cards, the child can arrange them in order.

ACTIVITIES

1. Invite five children to join you for some number activities. As they do so, distribute the numeral cards randomly on the floor. Ask each member of the group to select one card and hold it face forward against his chest. Ask the children to decide whether they are in the right order. See if they can arrange themselves by questioning who should be first?, second?, third?, fourth?, fifth?. When they are agreed on the correct order ask them to scramble themselves. This they may do by exchanging cards with someone else or by moving to another place. Let them take turns restoring proper order after each scramble.

2. Ask a member of the group to arrange the numeral cards in the correct order in a card holder. Choose one child to hide his eyes and another one to remove one of the cards, closing any space that is left. At a signal, the child hiding his eyes tries to identify the missing numeral and show where it belongs.

Content

NATURAL NUMBERS

Ordinal Use
First through Third
Last

Objective

Given a sequence of objects or people the child can identify those which are first, second, third, and last in the sequence.

ACTIVITIES

1. Use each opportunity to indicate, by touching, things that are in first, second, third, and last order as you work with children. These may be toy animals in a parade, children as they take turns for some activity, dolls arranged according to size, etc. As the children begin to use the language they should touch the object being identified.

2. Physical activities lend themselves to ordering. As children are arranged in short lines, they may be given directions similar to the following:

   a. All the children who are first will jump the long rope.

   b. All the children who are second will walk the walking board.

   c. All the children who are third will jump from this mark.

   d. All first players, give the ball to the second player in the row.

   e. All last players, exchange places with the first player in the row.

3. Arrange children who are going to do imitations in a line. Ask others in the group to guess what the first, the second, the third, and the last child is doing.

4. During story time ask

   a. What did the first Billy Goat Gruff say? the second? the third?

   b. What happened to the first Little Pig? the second? the third?

   c. Who did the Gingerbread Boy meet first? second? and so on to the last.

Content

Cardinal Use
The One-more Pattern
1 through 5

Objective

Given a set containing as many as five objects, the child can name the number of objects obtained by adding one more object to the set.

ACTIVITIES

1. In natural situations children will have many experiences involving one more, as

   a. When one more child has a birthday he joins the set of children who have already had a birthday this month and they are all older by one more year.

   b. A new baby joins the set of children in the family and there is one more in the set.

   c. When a child brings a doll or any other toy, there is one more added to the set.

   d. When a new member joins the group there is one more in the set of children on the rug.

   The following illustrates this teaching-learning situation.

   Teacher: Kerry, tell us how many nickels you had last week.
   Kerry: I had three nickels.
   Teacher: Did you get some more?
   Kerry: I earned one more. Now, I have four nickels.

Content

NATURAL NUMBERS
Cardinal Use
1 through 5
The One-less Pattern

Objective

Given a set of five objects, the child can name one less than the assigned number.

ACTIVITIES

1. As children play they have many opportunities to discover and use the one-less pattern.

   Invite several children to join you for some "number talk." In the area where this will take place provide two sets of toy animals, two sets of doll house furniture, and two sets of toy cars. Arrange the pairs of sets so there is always one more object in one set than there is in the other set. Have the children work in pairs and ask each child to compare his set with that of his partner as they "talk." Their conversation may, with your questions, develop in the following way.

(Continued on next page)

"What can you tell about your set, Gene?"
Gene: "I have one more animal in my set than Joe has."

"Can you tell how many you have?"
Gene: "Five."

"Good. What can you tell us about your set, Joe?"
Joe: "I don't have as many as Gene. I only have four."

The use of less than and one less will be established only after the children are given many opportunities to hear and use these phrases. Eventually Joe should be able to answer:

"My set of four animals is one less than Gene's."

2. Most children are familiar with "counting down." Practice the one-less idea by using such songs as:

   a. "Five Little Chickadees" during which children can remove themselves from the group as others sing.

Five little chickadees sitting in the door.
One flew away and then there were four.

Four little chickadees sitting in a tree.
One flew away and then there were three.

Three little chickadees looking at you.
One flew away and then there were two.

Two little chickadees sitting in the sun.
One flew away and then there was one.

One little chickadee left all alone.
He flew away and then there were none.

As different groups of children dramatize the "Five Little Chickadees," they may use numeral cards to identify the number of students remaining in the set as it decreases in size. This will provide an excellent opportunity to introduce the numeral card for zero at the conclusion of the song.

   b. "Five Little Ballons"

Five little balloons going up, up, up.	(Hold up 5 fingers and raise arm to a higher level each time you say "up".)
One went Pop ! ! !	(Clap Hands)
And then there were ______	(Children say 4 and hold up 4 fingers)
4 little balloons going up, up, up.	
One went Pop ! ! ! and then there were ____ !	(Children say 3 and hold up 3 fingers)

Continue the song until you reach, ". . . and then there were none." As children gain proficiency in counting they may begin with more than five balloons.

Content

NATURAL NUMBERS

Counting
1 through 30
1 through 99

Objectives

Given a set containing as many as 99 objects, the child can

1. count the members and
2. name the number of things in the set.

ACTIVITIES

1. Have children work in pairs. Direct one child in each pair to place a handful of things such as beans, jacks or rocks on a desk or the floor. Let the other child count the objects in the set. Repeat the activity using different numbers of things.

2. Start counting at some number such as 50. Then call on someone to repeat the counting and continue the sequence. That child may call on another until the sequence has reached a designated number, or a child might stop the series and begin another by saying, "New Game."

3. Your children might like to create poetry while counting. As they count two numbers at a time challenge them to make a rhyme. For example:

One, two, a cow says moo;
Three, four, lions roar;
Five, six, a monkey does tricks;
Seven, eight, who's swinging on the gate?
Nine, ten, a rusty safety pin . . .

Content

NATURAL NUMBERS

Ordinal Use
First through fifth
Last

Objective

Given a sequence of 5 objects or events, the student can identify the position in space or time of a particular object or event by saying: "This *object* is fourth" or "That *event* happened second."

ACTIVITIES

1. Give each child a piece of paper and direct him to trace around one hand. Call the thumb the first finger. Ask the class to draw the following on their tracings: a bandage on the second finger, a ring on the fourth finger, and a long fingernail on the fifth finger.

2. Have children identify themselves by ordinal numbers when they line up to go outside or leave for lunch.

3. Arrange a group of chairs to form a row or semicircle in the classroom and ask a child to sit in the chair between the third and fifth chairs. Ask the class to give another name for the last chair in the arrangement. (If there are five chairs, it is the fifth chair.)

4. Provide opportunities to discuss some of the following topics.

   a. The organization of a dramatic play: what is the first act?

   b. The calendar: what is the second day of the week?

   c. The sequence of tasks in an experiment or recipe: what do we do first?

   d. The naming of grades in school: first grade, second grade, etc.

   e. The names of teachers and the grades they teach: Miss Bailey, a third grade teacher.

   f. The repetition of a topic: "This is the second time we have discussed _________."

NUMBERS B-3

Content

WHOLE NUMBERS

Meaning of Zero

Objective

Given an example of the empty set, the student names its number as zero.

ACTIVITIES

1. Provide an opaque box, paper bag, or other container at the front of the room with several objects in it. If, for example, there are five blocks in the container, ask each of three children to remove one object. Have a fourth child tell how many are left and then show the remaining objects to the class to see if he is correct. Repeat this activity several times with the last child each time naming the number of blocks remaining.

   Then begin with two objects in the container and ask each of two children to remove an object. When a third child is asked to tell how many objects are left, if he says *none*, encourage him to use a number to describe the set of objects remaining. In this way the class will learn to use *zero* as the number of the empty set.

2. Draw several empty closed curves on the board. Show the class an empty paper plate or point out a table top which has nothing on it.

   Ask a child to name the number of objects drawn within the curve, or resting on the plate, or placed on the table. If he answers *none*, point out that this is the empty set whose number is zero. Ask another child to write the numeral for zero on the board.

NUMBERS B-4

Content

WHOLE NUMBERS

Grouping by Tens

Objective

Given ten ones, the child can name them as 1 ten.

ACTIVITIES

1. Use pennies and dimes and have the children trade 10 pennies for 1 dime, and 1 dime for 10 pennies.

2. Give each child 11-19 tongue depressors or other objects. Have them bundle 10 ones to make 1 ten. There will be some left over. Have them show that 13 objects is 1 ten with 3 left over.

Content

WHOLE NUMBERS

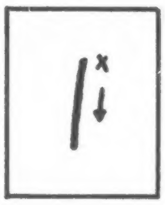
Numerals
0 through 9
0 through 31

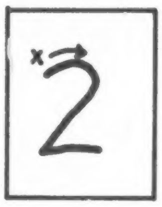
Objectives

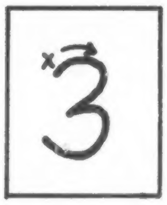
The student can
1. read and
2. write
numerals for whole numbers.

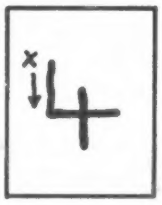
ACTIVITIES

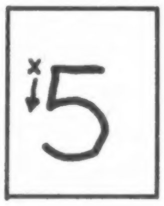
1. To help children write numerals correctly and without reversals, use a crayon to write any numeral causing difficulty on a 3" by 4" piece of sandpaper. Mark the place to begin with an "x" and draw a small arrow to show direction. The child can use his index finger to trace each numeral for kinesthetic reinforcement of the proper formation of these numerals.











It might be helpful for a child who writes many reversals to have a set of these sandpaper squares taped on his desk.

2. Identify and read numerals on calendars, lockers, rooms, pages, clocks, etc. Select and write some of these numerals. Record the daily temperature at certain intervals during the day. Keep the scores of games which involve and interest the children.

3. Duplicate a calendar with the name of the month, the days of the week and the first and last date of the month. Ask the children to fill in the missing numerals in the appropriate spaces.

Content

WHOLE NUMBERS

10 through 20

Objectives

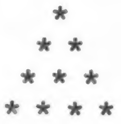
Given a set of ten or more objects the child can
1. name and
2. write
the number of the set.

ACTIVITIES

1. Show the students a set containing ten objects. Add one more to it. Ask them, "How many are there now?" Add another one to this set. "How many objects are in the resulting set?" This can be continued until the set has twenty objects.

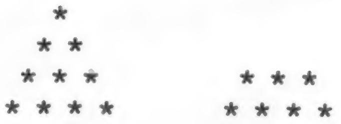
2. Tell the students that some mysterious person has arranged all the small objects in the room so that there are grouped

like this:



(10)

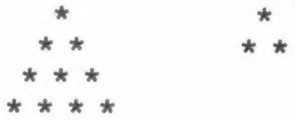
and this:



(10 + 7 = 17)

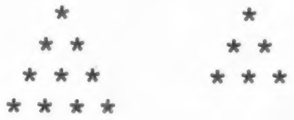
Ask them to look at sketches like the ones below and write the numeral for these objects:

a.



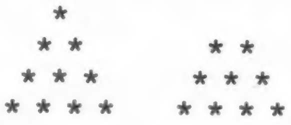
(13)

b.



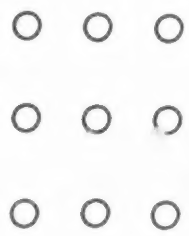
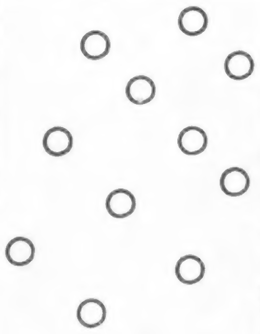
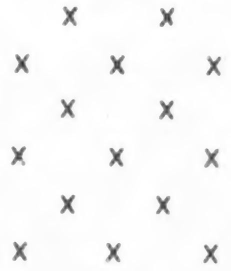
(16)

c.



(19)

3. Prepare the following table for use with your students. Draw several objects in each section of the first row. Have the students count them and write the number of objects in the space below each sketch as shown by the example.

Objects			
Number	(9)	(11)	(15)

Variation: List several numerals in the second row and have the students draw a corresponding set of sketches in the first row.

Content

WHOLE NUMBERS

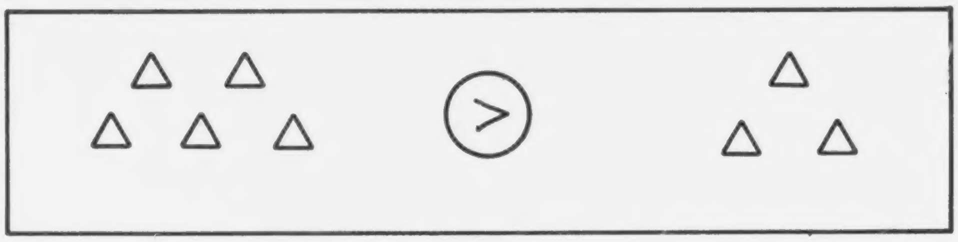
Order (<, >)
0 through 99

Objective

Given two whole numbers such as 13 and 29, the student can order them by saying "13 is less than 29" and by writing 13 < 29. Also, 29 > 13 is correct since "29 is greater than 13."

ACTIVITIES

1. Have the child write < or > between sets of objects that have been placed on his table or desk. For example:



In the diagram above, the five objects on the left represent a larger number than the three on the right. Therefore, the *is greater than* symbol is the correct answer.

2. Direct the children to write a column of numerals down the center of their papers. Then have them write a name for a number that is greater than the number named in the center on the right side of the original numeral and one that is less than it on the left side.

A Number Less Than	Original Number	A Number Greater Than
3	9	10
2	6	8
17	19	24

3. Use bundles of straws, sticks or pieces of paper to establish that in two digit numerals if the ten's digit of one numeral is larger than the 10's digit of another, you need not look at the rest of the numeral to determine which names the larger number. Dictate pairs of numbers and ask the children to make a true statement using the symbols, < or >. For example:

Dictate: "17, 23"

(Since 1 ten is less than 2 tens, 17 is less than 23. Thus, 17 < 23 is the statement you hope your students will be able to write.)

Content

WHOLE NUMBERS

Other Number Names

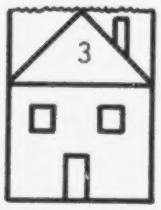
Objectives

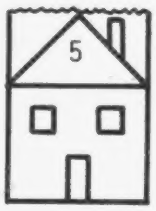
The student can
1. identify and
2. write

different numerals for the same number.

ACTIVITIES

1. Outline the picture of a house on a large paper bag. Make several of these and then number the sketches of the houses. Two of these are pictured below.



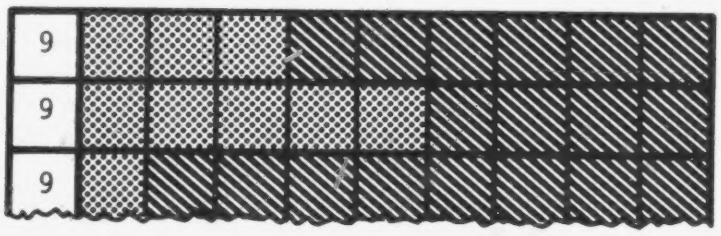


Prepare a set of flash cards with different names for the same number, e.g.,

3	5
1 + 2	3 + 2
3 + 0	6 - 1
4 - 1	1 + 4

Challenge the children to place the cards in the correct bag or "sort the mail" so that it gets to the correct house.

2. Duplicate copies of a sheet marked off in 100 sections, i.e., ten rows and ten columns. Write a 9 in the first column of each row. Then have the children make as many different combinations of 9 as they can by using two different colors each row.



How many of the ten rows will they use? Ask them to write the names for nine which they have illustrated. In the example above they would be: 3 + 6, 5 + 4, and 1 + 8.

This activity can be used to rename other numbers.

3. Give each child several objects, for example, 5 plastic discs. Have the students use these five discs to form combinations of two or more sets that make five. Record the various results. Some of these are: 2 + 3, 4 + 1, 1 + 4, 5 + 0, and 1 + 1 + 3.

Content

WHOLE NUMBERS

Words for 0 through 10

Objectives

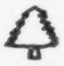
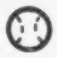
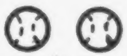
Given numbers from 0 through 10, the child can
1. read and
2. write
the number words.

ACTIVITIES

1. Have the children match pictures of sets and the corresponding number words.

2. Prepare a set of numeral flash cards and a set of number word cards for the numbers zero through ten. As the teacher or a child flashes a card have the class clap their hands the number of times indicated by the card. If several sets of these cards are available the children may use them for matching games.

3. Provide each child with a sheet of paper that has been marked off in rows containing 4 columns labeled as follows:

Set	Number Word	One More	Number Word
	Zero		One
	One		Two

Direct the children to draw sets in the first column whose sizes illustrate the numbers from zero through ten which you have written in the second column. In the third column have the class draw a set in each row that has one more member than the original set. In the fourth column write the number word for the new set. A couple of examples are given in the sketch.

4. Read stories that use number words. Individuals or groups within the class might create stories which contain number words.

NUMBERS B-10

Content

WHOLE NUMBERS

Skip Counting
by 2's, 5's, 10's

Objective

The child can count by 2's through 20, by 5's through 50, and by 10's to 100.

ACTIVITIES

1. While they are using a floor bead frame that has 10 wires with 10 beads on each wire, have pupils count 1 ten, 2 tens, 3 tens, 4 tens, etc.

2. Give pupils from 60 to 90 objects such as beans and ask them to make sets of ten. Then have them count the beans by tens.

3. Place cards with 10 buttons or x's on each card on a table. Distribute a card with one of the numerals 10, 20, 30, ..., 90 or 100 written on it to each of several children. Have each child bring his numeral card to the table, tell how many cards of buttons he needs to match his numeral card and then take that many back to his desk. After all the children with numeral cards have matched them, direct them to bring the cards back in order such as 1 ten, 2 tens, etc.

4. Use an abacus in counting by 1's, 2's, 5's and 10's. Move the beads to the left as you count and leave appropriate spaces between the groups of beads. This can either be teacher directed or done by individual children.

5. Arrange numeral cards showing counting by 1's, 2's, 5's or 10's in a random order on the chalk ledge. Have the children arrange the cards in the proper order.

NUMBERS B-11

Content

WHOLE NUMBERS

Place Value
1 through 10
1 through 99

Objective

Given sticks, straws or strips of paper, the student is able to demonstrate that 10 ones make a ten.

ACTIVITIES

1. Have a child show by moving one bead over at a time on an abacus that 10 ones make one group of ten. Have another child count a set of sticks or other objects until he has 10 ones grouped together.

2. Use colored discs and have white represent ones and red represent tens. Ten white can be shown to be the same as one red. Thus, 23 white discs can be represented as 2 red and 3 white discs.

(Continued on next page)

*3. Make some graph paper with 10 rows of 10 one inch squares to use in demonstrating the importance of ten as the base of our numeration system.

Have the children cut the paper into strips of ten squares each. Then cut some of the strips of ten into single squares. These single squares and strips can be laid on top of the hundreds sheet to show that 10 ones make a ten and 10 tens make a hundred.

NUMBERS B-12

Content

WHOLE NUMBERS

Place Value
Ones
Tens

Objectives

Given a numeral such as 43, the student can
1. name and
2. distinguish

the numerals that are in the ones and tens places.

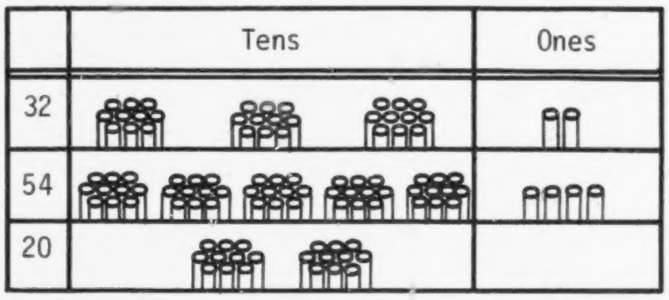
ACTIVITIES

1. Provide each child with a supply of sticks, straws or strips of paper. Ask the children to group them in sets of ten and bundle these with rubber bands. Leave some of the items unbundled. Write a two digit numeral on the chalkboard and have the children represent this by using the counting aids.

2. Use oral exercises such as "I am thinking of a number that has 4 tens and 6 ones. What is it?"

   Variation: "Tell me how many tens and ones are in the number 50."

3. Duplicate sheets similar to the following:



Have the children draw pictures for each given numeral using a bundle symbol for ten and a stick symbol for one.

NUMBERS B-13

Content

WHOLE NUMBERS

Expanded Notation
11 through 99

Objective

Given a numeral such as 54, the student can write the expanded numeral: 50 + 4

ACTIVITIES

1. Have a group of 10 children come to the front of the room. Ask another set of 5 children to join them at the front of the room. Write the numerals representing these sets on the chalkboard in this manner, 10 + 5 = 15.
2. Discuss the page numbers found in books and have the students write some of them in expanded notation such as page 50 + 4 for page 54.

NUMBERS B-14

Content

FRACTIONAL NUMBERS

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

Objectives

Given a model for one half, one third or one fourth, the student can
1. identify and
2. name

the fraction associated with the model.

ACTIVITIES

1. Prepare and distribute sets of fraction cards with each set containing cards with the numerals for one half, one third or one fourth written on them. Display several objects such as an apple, a piece of paper, a piece of string and a strip of paper. Cut these items into halves, thirds, or fourths. As you cut an object and display one of its parts, have the children show the cards that name the fractional number associated with the model.
2. Prepare a set of cards showing drawings of fractional parts of objects and another set showing the numerals for one half, one third and one fourth. Have the children match the drawings with the correct numeral cards.
3. Ask the children to place 10 discs or other objects on their desks, and then instruct them to pick up one half of them. Use the same type of activity with different numbers of objects to illustrate one third and one fourth.

Content

FRACTIONAL NUMBERS

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

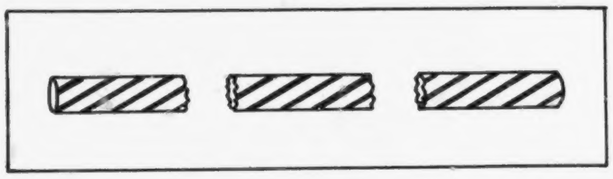
Objectives

Given appropriate materials such as paper, felt regions, or counting rods, the student can
1. identify or
2. construct
models for one half, one third, and one fourth.

ACTIVITIES

1. Encourage the children to create stories with problem situations to tell each other. Have them draw pictures to illustrate their answers. For example:

Philip has a stick of candy. He decides to share it with two friends. Draw a candy stick and show how Philip can break it into three pieces of the same size.



2. Provide each child with a rectangular sheet of paper. Direct the children to fold the sheets in half. Discuss the two halves and what each might be called. The paper can then be folded to show fourths, and then these parts can be named.

3. Use edible objects to give children additional experiences in working with fractions. Have one child do the dividing and other children select the fractional parts they wish to eat. You might require the children to identify the fractional part they select before they are given permission to eat it. Under these conditions the parts will very likely be close to the same size!

Content

WHOLE NUMBERS

Even and Odd
0 through 30

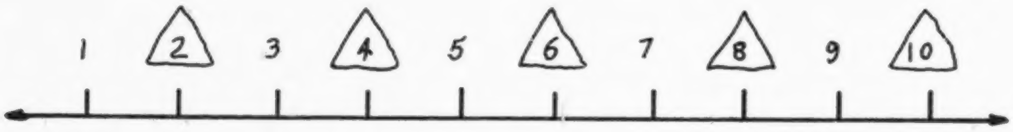
Objective

Given a set of objects, the pupil can distinguish whether there are an odd or even number of objects in the set by arranging them in a pattern.

ACTIVITIES

1. Write several numerals on the chalkboard such as 5, 11, 18, 20 and 15. Instruct the children to use discs to illustrate each number on their desks. Direct them to place two discs in each row as if they were counting by 2's. They will discover some numbers form sets with a counter left over. These are odd numbers. If all counters are paired, the number is even.

2. Duplicate a number line with ten equally spaced points marked on it. Starting with the second point from the left and continuing with every other point draw a triangle above each of these points. Distribute to the children and have them write the numerals for the even numbers in these triangles. Then ask them to write the numerals for the numbers represented by the remaining points that are located on the number line. The finished effort might look like this:



As a variation of this activity locate the triangles so that the student records the odd numerals in the triangles and the even numerals between them.

Content

NATURAL NUMBERS

Ordinal Use
First through
Twelfth

Objective

Given a sequence of objects or events the student identifies the position in space or time of a particular object or event by saying: "This object is ninth from the right" or "This is my seventh birthday."

ACTIVITIES

1. Arrange the children in rows with no more than twelve in a row. Ask each child his position in his row. Answers will vary from first through twelfth.

   Play *Simon Says* using directions such as these:

   (a) Simon says for the fifth child to touch his toes.

   (b) Simon says for the third child to jump.

   Continue until each child has had an opportunity to participate.

2. Divide the class into teams and give each team 12 small containers, such as jar lids. Ask each team to sit in a circle on the floor and to line up the 12 containers in a straight line. The child who is "It" should place a small object in one of the containers and call on another child to describe what he has done. If he puts the object in the third container, the child he calls on should say, "Don put the object in the third container." If the child answers correctly, he becomes "It" and the game continues.

NUMBERS C-2

Content

WHOLE NUMBERS

Counting by
1's, 2's, 5's
and 10's

Objective

The child can count by

1's through 1,000,
2's through 100,
5's through 100 and
10's through 1,000.

ACTIVITIES

1. Provide a large quantity of small objects so students can actually group things as they count. Buttons, cubes, beans, bottle caps, and counters are useful for this.
2. Have a child begin counting. After he has counted a short while, ring a bell to signal him to stop. Then point to someone else to continue the counting where the first child has stopped. Try this same activity when counting by 5's or 10's.

NUMBERS C-3

Content

WHOLE NUMBERS

0 through 999

Objectives

Given whole numbers from zero through 999, the child can
1. read and
2. write
numerals for them.

ACTIVITIES

1. Use the element of chance to provide interest and excitement while your children are accomplishing this objective. You will need two items for this activity: (1) a random arrangement of numeral cards containing those numbers you wish to cover (2) a container with name cards for all the children present.

   Begin by drawing the name of a student from the container. Have that child come to the front of the class, select a numeral card, and read it aloud to the class without showing it to the other children. They will each write a numeral for that number on a sheet of paper and then compare this numeral with the one on the card when it is shown to the class. This card may then be set aside and the child can draw the name of another student who will select the next numeral card.

   This activity lends itself to large or small group involvement and can easily be done in the few remaining minutes before lunch or at the end of the school day.

Content

WHOLE NUMBERS

Order
0 through 99

Objective

Given two whole numbers, such as 12 and 14, the student can name the number between them.

ACTIVITIES

1. Provide each child with a sheet of ditto paper or construction paper drawn off in squares with the numerals 1 through 50 written in them. Have the children cut out the 50 squares and put them in an envelope.

   Select two numbers such as 12 and 14 and write their numerals on the chalkboard or overhead projector. Have the children shuffle their cards and then locate the one containing the numeral which comes between 12 and 14. (13)

   Continue this by asking individual children to select other pairs of numbers for the class to consider.

   After they have completed this activity, instruct the children to place the squares in numerical order, fasten them with a paper clip, and return them to the envelope. In this way much time will be saved when they are used for future activities.

Content

WHOLE NUMBERS

Order
0 through 99

Objective

Given a number between 0 and 99, the student names the numbers which come before and after it.

ACTIVITIES

1. A number line can be used in a variety of ways to help students accomplish this objective. It may be drawn on the chalkboard, on the overhead projector, placed on a felt board, printed on paper and distributed to the children, or drawn on paper and taped to the floor. Teachers may use it with individual students or the entire class. Points on the number line may or may not be labeled.

   Use the number line to locate some point. Name it. Then ask a student to name the point preceding it. In the case of the number line on the floor, a student might be asked to step backward one unit and name his new position.

(Continued on next page)

The next question will be to name the point on the other side of the original point selected. For the student standing on the number line on the floor, this will mean identifying his new position after moving forward two units.

2. The use of chance offers an interesting way to have students work with consecutive numbers and the ordering of numbers.

   Make a set of numeral cards from 0 through 99 and place them in a container. After mixing them thoroughly have students volunteer to randomly select a card from the container and give the numbers which come before and after the number written on the card.

   A good sequel to this activity is to have the students keep their cards and after several have had turns, ask them to arrange themselves at the front of the classroom in numerical order from smallest to largest.

NUMBERS C-6

Content

WHOLE NUMBERS

Skip Counting
by 2's, 3's, 4's,
5's and 10's

Objective

The pupil can write numerals through 50 that result from counting by twos, threes, fours, fives and tens.

ACTIVITIES

1. Use the numeral cards prepared in Numbers C-4. Direct the children to place their numeral squares across the tops of their desks so they can easily see them. Give these directions for arranging the numerals:

   a. Show me the numerals you would use to count by threes up to 50.

   b. Determine the numerals you would use to count by fours up to 50.

   *c. Locate the numerals you would use to count by sixes up to 50.

   The teacher can walk around the room and readily spot those children who are having difficulty.

2. Directions such as those given above could be prepared on sheets of tagboard and filed in an Arithmetic Activities box for children to use independently. Answers can be written on the other side of the tagboard so that the activity can be self-checking.

Content

WHOLE NUMBERS

Word Names for
0 through 20

Objectives

Given a whole number from 0 through 20,
the child can
1. read and
2. write
the word name for it.

ACTIVITIES

1. Prepare several sets of number word and numeral cards out of tagboard for the whole numbers from zero through twenty. Have the children match each number word with the correct numeral. When several sets are available the children will enjoy using them for independent activities. If you provide an answer key for the number words and their numerals the children can check their matching.

2. Distribute number word cards to part of the class and numeral cards to the other children. Have each child find his partner. Discuss any errors that might occur.

   Variation: When the children are not present place the numeral cards in fairly obvious places around the room. Then, during class, give each child a number word card and have him find his matching card.

Content

WHOLE NUMBERS

Place Value
- Ones
- Tens
- Hundreds

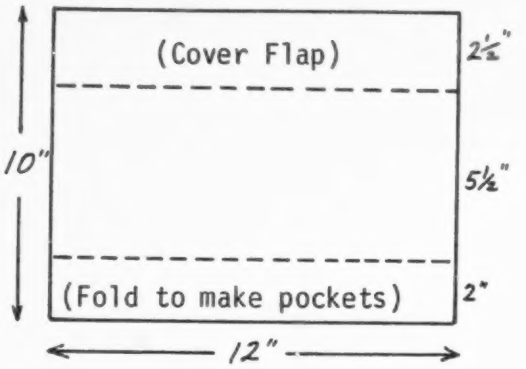
Objectives

Given a three-digit numeral, the student can
1. name and
2. distinguish

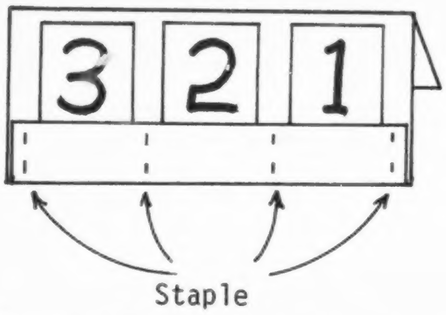
the digits that are in the ones, tens, and hundreds places.

ACTIVITIES

1. Help the children make place-value charts for themselves by folding pieces of tagboard like this:



Completed chart with cards indicating 321.



Sample card

Have the children make three sets of 3" x 5" numeral cards. Each set is to consist of ten cards for the numerals 0 through 9. The use of three different colors will help keep the sets separated. Ask the children to arrange the sets in numerical order for use in each pocket of their place-value charts.

Ask each child to form a three-digit numeral using his numeral cards and place-value chart.

Have a child explain what the individual digits of his numeral represent. (321, for example, represents 3 hundreds, 2 tens, and 1 one.)

Ask him to use those same three digits to represent a different number. (312, 231, 213 are some possible answers if 321 is the original numeral.)

How many different numbers can be formed using three different digits? (6)

*Can the children think of a three-digit numeral whose digits name only one number no matter what sequence they are in? (111, 222, 333, etc.) Can they give an example of one whose digits can be rearranged to name only three different numbers? (122: 122, 212, 221)

NUMBERS C-9

Content

WHOLE NUMBERS

Place Value
Role of Zero

Objective

Given a numeral which contains a zero as one of its digits, the child demonstrates his understanding that the zero means *not any ones* or *not any tens* according to the place in which it appears.

ACTIVITIES

1. At this level students will likely study, at most, three-digit numerals. The presence of zero in the ones place indicates that after the removal of sets of ten from a group of objects there are none left over to be counted.

   Such items as counters, ice cream sticks, soft drink caps, pencils, or pipe cleaners might be used for the following grouping activity. Have the students bundle or group these objects into sets of ten. Students should have many opportunities to do this involving quantities of objects which are not exact multiples of ten. Have the children name the number of objects they have grouped as "4 tens" in the case of forty objects, or "3 tens and 7 ones" for thirty-seven. You may then wish to ask them to write the numeral. For the two examples just given they would write 40 and 37.

   The important point here is to have the students explain that the zero in 40 indicates that after the grouping of that many objects by tens, there are no ones left over.

NUMBERS C-10

Content

WHOLE NUMBERS

Order (<, >)
0 through 999

Objective

Given two numbers such as 213 and 347 the student can order them by saying, "213 is less than 347," or "347 is greater than 213" and by writing, "213 < 347" or "347 > 213."

ACTIVITIES

1. On 4" x 6" cards write large numerals selected from the set of whole numbers from 0 through 999. Shuffle the cards and hold them so that the numerals are not visible. Ask several children each to select a card and arrange themselves so that their numbers are in sequence from smallest to largest. Have them face the class and display the cards. The class should judge whether the ordering is accurate. Allow one child to read the numerals. If there are any errors have the class offer corrections. These children can then have other children join them and everyone in the group can select a new card. This will give more members of the class opportunity to participate directly and provide a larger set of numbers to be ordered.

(Continued on next page)

For a variation have the children line up in descending numerical order. Observe how long it takes them to realize that they can line up the same way as before and read starting from the other end of the line, i.e., beginning with the largest number.

2. Select at random two of the cards and place them on the chalk tray for everyone to see. Have a student go to the board and write the appropriate symbol, either < or >, between the two cards. Ask him to explain his choice of symbols.

NUMBERS C-11

Content

WHOLE NUMBERS

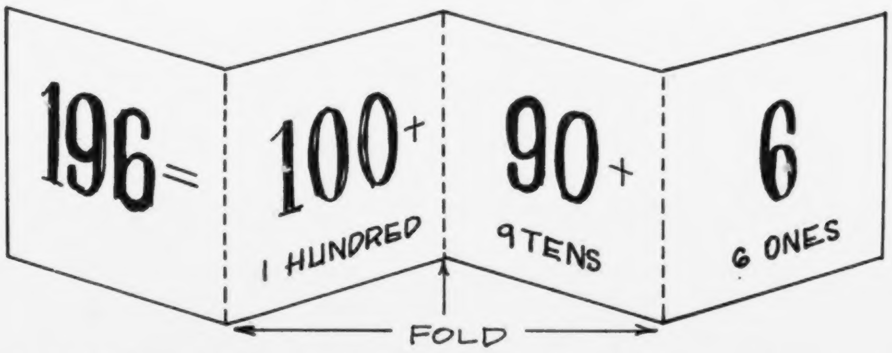
Expanded Notation
Place Value

Objective

Given a three digit numeral such as 196, the child can write the expanded numeral:
1 hundred + 9 tens + 6 ones or 100 + 90 + 6
and vice versa.

ACTIVITIES

1. Prepare cards on 5" x 18" strips of manila paper to show the expanded form of numerals, for example:



If these cards are folded as shown in the illustration above, the standard form of the numeral will appear on the top. After a child has written the expanded numeral he can open the card to see if his answer is correct.

Content

WHOLE NUMBERS

Renaming
0 through 999

Objectives

Given a number such 123, the child can
1. identify
2. read and
3. write

different names for this number, for example:

$$123 = 10 + 20 + 3$$
$$= 130 - 7$$

ACTIVITIES

1. Prepare two boxes, one red and one blue, and place them along with ten objects such as checkers on the table. Have the children put these checkers in the boxes in several ways and keep a record of these arrangements, for example:

	Red Box	Blue Box
First Student:	6 checkers	4 checkers
Second Student:	5 checkers	5 checkers

After creating an arrangement, a child might then write the renaming of ten he has illustrated. The following number sentences contain the names illustrated by two examples above.

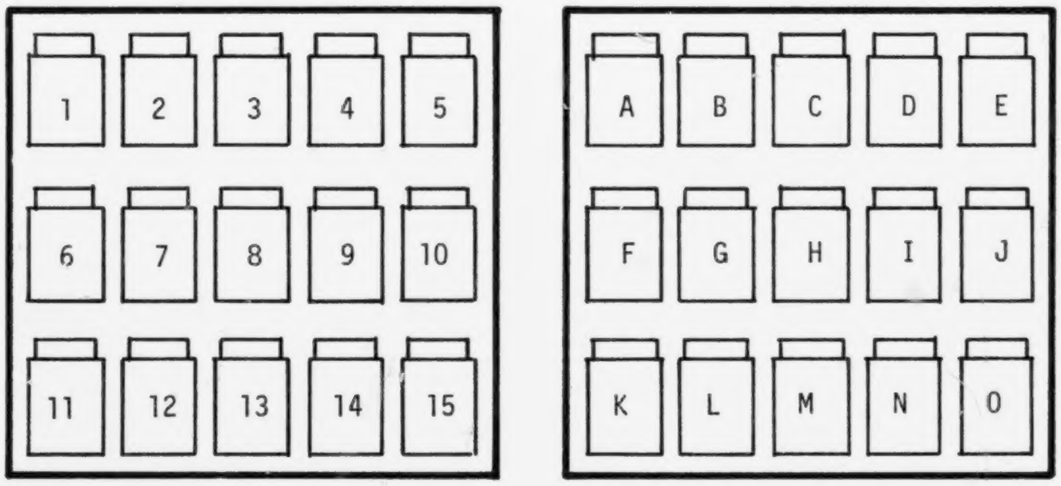
$$6 + 4 = 10$$

$$5 + 5 = 10$$

Have the children find as many arrangements as they can. Check each one against the record to see if it is different.

You may wish to include a third box. Color it a different color, perhaps yellow or green. Three boxes permit the following kind of sentence to be written: 2 + 3 + 5 = 10

2. Prepare two sets of index cards, one representing numbers as 60 + 4 and the other showing it as 64. On each of two sheets of brightly colored posterboard, glue 15 library card pockets in 3 rows of 5 each. Label one set of pockets with the numerals from 1 through 15, and the other set with the letters from A through O.



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Shuffle one set of cards and place them face down in the pockets of one sheet of posterboard; then shuffle the second pack and place those cards in the pockets of the other sheet so that their numerals also cannot be seen. Hang the two posterboards where they are easily accessible to the children.

Direct a child to choose a card from a pocket on each sheet. Have the child read the two numbers. When they match as in the example of 60 + 4 and 64, have him remove the cards. This combination scores a point. The pairs of cards which do not match are to be returned to the pockets. These basic sets of cards can be changed to review many skills and concepts.

3. Give each child a sheet of newsprint. Have them fold the paper so they will each have four sections of equal size. Next have them write four numerals on one side of the sheet, one numeral in each section. Now direct the children to write as many other names for these numerals as they can. By placing a time limit on this activity you may make it more challenging. The children can then exchange papers and check each other's work. Do not discourage children from employing any pattern which is correct.

NUMBERS C-13

Content

FRACTIONAL NUMBERS

Halves
Thirds
Fourths

Objectives

Given a model of a fractional number, the child can

1. identify,
2. write, and
3. read

the fraction associated with the model.

ACTIVITIES

1. If you have solid, take-apart models this is an excellent opportunity to use them. Otherwise, you can illustrate a number of geometric shapes or objects on the chalkboard or other visual aid. Use a dotted line to show such parts of a whole as halves, thirds, and fourths. Have the children decide how the whole is divided and write the appropriate fraction.

   Actual objects divided into parts will illustrate this more effectively than pictures or sketches. You and your students may wish to prepare a fraction kit similar to those available commercially.

Content

FRACTIONAL NUMBERS

Halves
Thirds
Fourths

Objectives

Given materials such as paper, felt regions, or number rods, the child can
1. construct and
2. identify
models for fractional numbers.

ACTIVITIES

1. Using actual objects such as wooden sticks, candy bars, sheets of construction paper, and crayons, show children the whole of each, then show them the same objects divided into halves, thirds, or fourths. Have the children investigate in groups with each group working at a station where these objects are located.

   After the class has had varied experiences with these objects, provide each child with two strips of construction paper of equal length. Have them cut one strip into thirds. Using similarly shaped flannel cutouts on the flannel board have the children illustrate such parts as one third, two thirds, and three thirds. One child might display the model on the flannel board as another child names the fractional number associated with the model. Children at their desks can show the fractional number by using their individual strips.

Content

NATURAL NUMBERS

Ordinal Use
First through
Thirty-first

Objective

Given an object or event, a child can identify its position in space or time by saying: "Today is the seventeenth" or "This is my eighth birthday."

ACTIVITIES

1. Use the children's addresses to discuss how the house number indicates the location of their house on the street.

   Have the class construct small model houses, label them with numerals and place them in their appropriate locations on a map drawn on the floor. This could be correlated with work being done in social studies.

2. Use the calendar to teach the ordinal use of numbers. For example: November 7 means the seventh day of November.

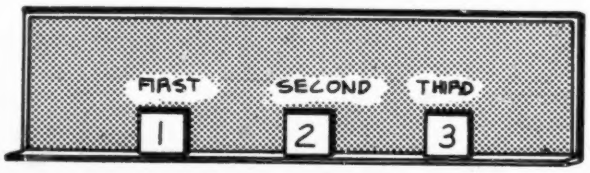
3. Give each child a position in line and a numeral card denoting his position. Then ask the children to scatter themselves throughout the room. When a whistle or signal is given have them arrange themselves in numerical order again.

4. Use this activity after your children have had many experiences with the ordinal use of numbers. It will help you determine the degree of their understanding of this use of numbers.

   Prepare a set of numeral cards from one through the number of children in the room.

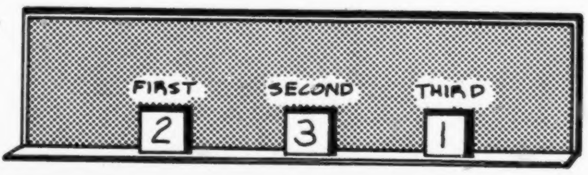
Natural Order with Ordinal Names

a. Place these on the chalk tray in natural or counting order. Then discuss the position of each numeral as a member of the set. Write the ordinal word name of each number.



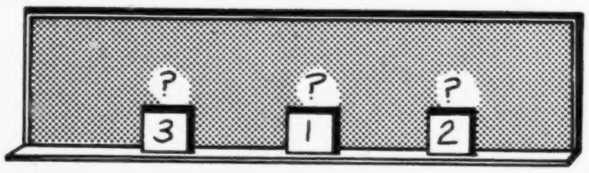
Shuffled Order with Ordinal Names

b. Rearrange the cards on the tray in any order. Discuss the fact that the ordinal word names of the numbers written above the cards did not need to be changed.



Reshuffled Order with No Ordinal Names

c. Erase the ordinal word names, rearrange the cards again in any order and see if the children can give the ordinal position of each card.



Content

WHOLE NUMBERS

Counting
Skip Counting by 2's, 3's, 4's, 5's, 10's and 100's

Objective

The child is able to count by 1's, 2's, 3's, 4's, 5's and 10's from least to greatest and in reverse order.

ACTIVITIES

1. Direct the children to write numerals for the even numbers in the appropriate squares of a ten by ten grid. Have them begin at the upper left and count across as shown below. They will, of course, be counting by twos.

	2		4		6		8		10
	12		14		16		18		20
	22		24		26		28		30
	32		34		36		38		40
	42		44		46		48		50
	52		54		56		58		60
	62		64		66		68		70
	72		74		76		78		80
	82		84		86		88		90
	92		94		96		98		100

A grid like this might be prepared on tagboard and children could use numbered tags to place the correct numerals inside the squares.

2. Prepare transparencies of the 100 squares with overlays to shade the areas covered when counting by 2's, 3's, 4's or some other number. Graph paper can also be used with shading to show the counting by various numbers. This will produce some interesting patterns.

Content

WHOLE NUMBERS

Reading, Writing
0 through 999,999

Objectives

Given whole numbers from 0 through 999,999
a child can
1. recognize
2. read and
3. write

numerals for them.

ACTIVITIES

1. Ask the children to bring in clippings from newspapers and magazines containing references to numbers greater than 1000 and less than 1,000,000. You may wish to have them read the sentences containing the numerals.

2. In the cross number puzzle below number words are supplied and the children are to write in the numerals.

1	2	3		▨
4			▨	5
6		▨	7	
▨	8			

Across

1. eight thousand, three hundred ten
4. nine hundred fifty-two
6. ninety-seven
7. sixty-four
8. four thousand, five hundred forty-one

Down

1. eight hundred ninety-nine
2. three thousand, five hundred seventy-four
3. twelve
5. three hundred forty-one
7. sixty-four

Children will enjoy making these puzzles as well as solving them.

Variation: Another type of cross number puzzle gives the numerals and has the child supply the words.

3. See Numbers D-6.

Content

WHOLE NUMBERS

Order
0 through 10,000

Objectives

Given numbers from 0 through 10,000 a child

1. names the numbers which precede and follow them, and
2. arranges non-consecutive numbers according to increasing or decreasing size.

ACTIVITIES

1. Prepare a non-consecutive set of numeral cards such as:

701	730	715	751	743

a. Shuffle the cards and have a child select one. Have him show it to the class and name the numbers which come before and after the number indicated on the card.

b. Place the cards in stacks of four or five. Divide the class into groups and give each group a set of cards. Have a group leader give each child a card. When it is his group's turn, he might direct the members of his group to arrange themselves in numerical order from least to greatest. Encourage the rest of the class to discuss the correctness or incorrectness of the arrangement. Continue until each group has participated.

Variations for labeling the cards are shown by the following examples:

300 + 2 + 7	20 + 7	Twenty	200 + 20	Thirty-two

Your children might enjoy making the cards themselves.

Content

WHOLE NUMBERS

Order (<, >)
0 through 10,000

Objective

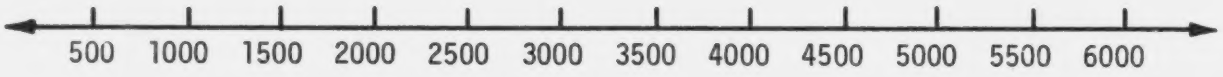
Given a set of whole numbers, a child can use < or > to show the order between any two of the numbers.

ACTIVITIES

1. Use a set of numeral cards such as those described in Numbers D-4. A child can randomly select two cards from the set and write their numerals on the chalkboard with the appropriate symbol between them. For example: 730 < 751.

   Another child may then read what the first has written: "730 is less than 751."

2. Use the number line to decide which of these symbols, < or >, to write between each pair of numerals below.



(a) 500 < 1000

(b) 3500 > 3000

(c) 3567 < 3657

(d) 1492 > 1176

Note: It is helpful to realize in example (c) that 3567 < 3657 because 3500 < 3600.

Many exercises can be designed using the number line to illustrate the *greater than* and *less than* relations.

NUMBERS D-6

Content

WHOLE NUMBERS

Words
0 through 999

Objectives

A child can
1. read and
2. write

number words for numbers up through nine hundred ninety-nine.

ACTIVITIES

1. Make a set of cards with word names on one side and numerals on the other. To be complete for numbers up through 999, this set should contain cards for 1-9, 11-19, and multiples of 10 through 90 and 100 through 900.

   The number 237 is illustrated in the following two ways.

200	30	7
two hundred	thirty	seven

   These cards can be used in a variety of ways.

   a. The student can be asked to write the simplest Arabic numeral. (237)

   b. He can be shown the front of the cards and asked to write the word name which he can then check for himself by looking at the back of the cards.

   c. The word name can be displayed and the child can be asked to write the expanded numeral which he can then check by turning the cards over.

2. See Numbers D-3.

3. Prepare one set of cards containing some of the more difficult number words from 0 through 999 and another set of the corresponding numerals. Children can use these in matching activities. The cards may be duplicated for card games. Two names for the same number can be called a book. The student who collects the greatest number of books is the winner. The cards can be used for Fish, Battle, or games which you and your students may invent.

Content

WHOLE NUMBERS

Place Value
Ones, Tens, Hundreds, Thousands

Objectives

Given a four-digit numeral, the child can
1. identify and
2. name
the place value of its digits.

ACTIVITIES

1. Use a make-believe newspaper clipping similar to the following. If possible, duplicate copies and distribute one to each child.

   > *Last Saturday 836 people visited the Central City Zoo. There were 683 children among the visitors. At noon, lunches were served in the lunch room to 386 of the visitors.*

   Ask the following questions and record the answers for the students to view on the chalkboard or overhead projector.

How many people visited the zoo?	*(836)*
How many of them were children?	*(683)*
How many lunches were served?	*(386)*

   Lead the students to observe that the same digits, 3, 6, and 8, appear in each numeral. Then have the children react to this assertion:

   > *Since the same digits appear in each numeral, the three numerals are just different names for the same number! So, all the people who visited the zoo were children and they all ate lunch in the lunch room at noon.*

   Children's reactions to this fallacious statement should naturally lead to the fact that the position or place of a digit in a numeral is very important - that 836, 683 and 368 are names for different numbers; that 8 in 836 refers to 8 hundreds or 800, that 8 in 683 refers to 8 tens or 80, and that 8 in 368 refers to 8 ones or 8. Similar statements may be made about the digits 3 and 6.

2. Divide the class into groups of 3 children each, and give each group a set of 4 different digits. Direct them to write as many different four-digit numerals as they can using their set of digits.

Example:	*Possible Numbers:*
3, 4, 5, 6	*3456, 3564, 3546, 3465, etc.*

   It is possible to write 24 different four-digit numerals if zero is not one of the digits used. Have each group share their numerals with the rest of the class. Those groups who found all twenty-four numbers might have used a pattern in naming them which they can share with the rest of the class.

Content

WHOLE NUMBERS

Expanded Notation
Place Value

Objective

Given a four-digit numeral, the student can write it as an expanded numeral, e.g., 2647 = 2000 + 600 + 40 + 7.

ACTIVITIES

1. Use flannel numerals with a flannel board to show an expanded form of 4-digit numerals. These can be arranged in 2 columns using a string or yarn to match those which name the same number. The children may then mix and match them again.

2. Prepare cards like the following for several 4-digit numerals.

1215

one thousand two hundred fifteen

1000 + 200 + 10 + 5

Shuffle the cards and have children arrange them in sets consisting of different names for the same number.

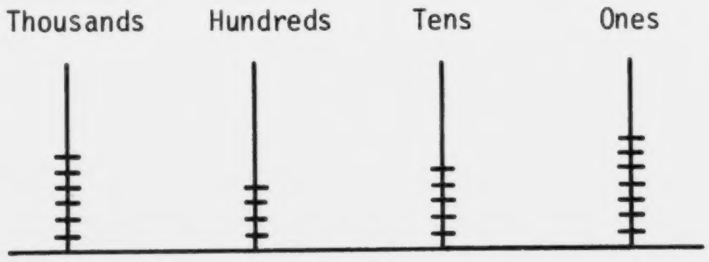
3. Prepare twice as many three-by-twelve inch tagboard cards as you have children in your classroom. On half of them write four-digit numerals; on the others express the same numbers using expanded notation, for example:

5623

5000 + 600 + 20 + 3

Divide the class into teams giving every team member a card showing the numerals in expanded form. Place the other set of numeral cards in the chalk tray, so they will be easily seen. Have the children get in relay formation and at a given signal one member of each team will race to the chalk tray, find the card that matches his card, and then return to the end of the line. When the entire team has finished the leader checks the pairs to see if they are correct.

4. An abacus on a transparency for use with an overhead projector allows you and the children to represent numbers which can be seen by the entire class. The number illustrated in the diagram to the right is 6457.



5. Dictate numbers for the children to write. Later have them read their numerals and interpret the place value of each digit, for example:

1245 is 1 thousand + 2 hundreds + 4 tens + 5 ones.

Content

WHOLE NUMBERS

Renaming

Objectives

Given a number, the child can
1. recognize,
2. read and
3. write

many different names for the same number, for example, 912 = 900 + 10 + 2 = 900 + 12.

ACTIVITIES

1. Assemble several boxes and pieces of cardboard cut in a variety of shapes such as a circle, a triangle, a square, a hexagon, and a rectangle to use as covers for the boxes. Rearrange the boxes on a table placed next to the chalkboard. On the chalkboard draw a shape similar to each box cover, e.g., ○ □ △ . Now place the box whose cover matches it under each shape drawn on the chalkboard.

   Count out some objects such as 9 blocks and place them in the box on the left which has the circular cover. Ask a child to lift the cover and count the objects. When he finds that there are 9, ask him to write the numeral 9 inside the circle on the board. (⑨ □ △) Ask the children to close their eyes. Tell them that all 9 objects are going to be taken out of the box on the left and put in other boxes and covered. Have the children open their eyes and ask a child to look in the box with the round cover. Ask: "How many are there now?" (None)

   Have someone else look in one of the other boxes. He might find 3 blocks in the box with the square cover. Ask him to record this inside the square. (⑨ $\boxed{3}$ △) Question: "How many are there in the other box?" (6) Write this inside the triangle. (⑨ $\boxed{3}$ 6⃤) Ask the children to write a mathematical sentence to describe this relationship. (9 = 3 + 6)

   Start this activity again with the same 9 objects. One of the children can play the role of the teacher and move the objects. Repeat this activity to show different names for the same number.

2. Use exercises like the following for enrichment in writing different names for the same number.

   a. Give a child a stack of cards containing many names for several different numbers. Instruct him to match the different names for each number in his stack.

   b. From activities like those above a child should realize that a number such as 5382 can be named in many ways. Ask him to write some of these names. In response to this he might list:

   5382 ones
   5000 + 300 + 80 + 2
   5300 + 80 + 2
   538 tens + 2 ones
   53 hundreds + 8 tens + 2 ones

Content

FRACTIONAL NUMBERS

Halves through Tenths

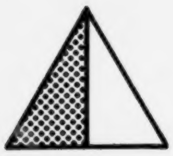
Objectives

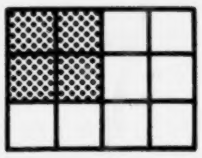
Given a model of a fractional number, the student
1. reads and
2. writes

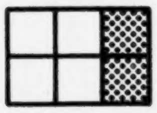
the fraction associated with the model.

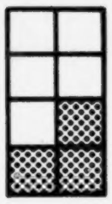
ACTIVITIES

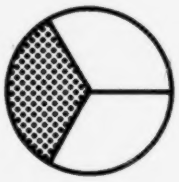
1. Prepare an overhead transparency of the following models of fractional numbers.


(a)


(b)


(c)


(d)


(e)

Have the children write the fraction associated with each model. These can be written on small tagboard cards which can be held up by the children so the teacher can quickly determine those who need assistance. The cards can be saved and used for similar activities at a later time.

2. Arrange various sets of children, chairs, books or other large objects in front of the room. Group members of a set to show parts of the set. Have the children write the fraction associated with each part.

Sets of coins or discs can be used on the overhead projector for the children to construct models of fractional numbers. One child may group the members of the set into distinct parts. Another child can write the fraction associated with this model.

Content

FRACTIONAL NUMBERS

Order
<, =, >

Objectives

Given models of two fractional numbers which can be named with fractions having the same numerators or the same denominators, the student can
1. name the fractions and
2. write the symbols <, =, or >

to express the relationship between them.

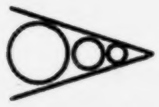
ACTIVITIES

1. It is difficult for some youngsters to comprehend that one third is greater than one twelfth since, after all, 12 is larger than 3. A good way to illustrate this is to have each student measure and mark a section one inch in length on a pencil. Then wrap a piece of rope around the pencil between these two marks and count the number of complete turns required to cover the distance. Follow this by wrapping the same distance with lightweight string. If it took only 3 turns of rope but 12 turns of string to cover the same distance, the students should more easily see that

$$\frac{1}{3} > \frac{1}{12}$$

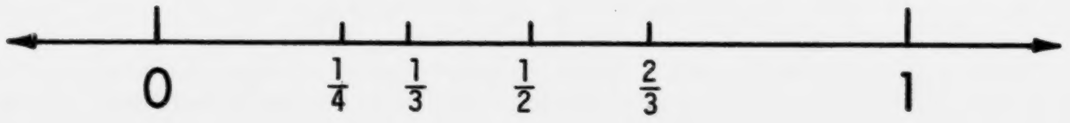
since the thickness of the rope is obviously greater than the thickness of the string.

One way to remember what the symbols < and > represent is to visualize circles drawn within an enlarged symbol, for example:



As you look at these circles, the first is larger or greater than the second. The symbol > means *is greater than.*

2. A number line provides a good visual link for comparing the values of fractional numbers.



When points are carefully plotted and it is remembered that values get larger in moving to the right along the number line, statements like those which follow can be more easily completed.

$$\frac{1}{3} \bigcirc\!\!\!\!\!< \;\frac{2}{3}$$

$$\frac{1}{4} \bigcirc\!\!\!\!\!< \;\frac{1}{3}$$

$$\frac{2}{3} \bigcirc\!\!\!\!\!> \;\frac{1}{2}$$

$$\frac{2}{4} \bigcirc\!\!\!\!\!= \;\frac{1}{2}$$

Content

FRACTIONAL NUMBERS

Halves through Tenths

Objectives

The child can
1. identify or
2. construct
models for fractional numbers.

ACTIVITIES

1. Prepare strips of paper of the same length for the class. Have each child fold one strip to make the ends meet, crease it and then open it. How many parts has it been partitioned into? (2) Now have them cut along the fold and label each of the parts. Write this name, one half, on the chalkboard.

   Show them a much longer strip of paper and ask a child to fold it into halves. Develop the idea that all halves do not look alike, but that each is half of the whole object which was folded.

   Give the children paper rectangles, squares, and circles. Ask them to fold each shape into two parts the same size and label each part. Emphasize the concept that two halves of the same object make one whole object. Continue with other fractional parts and with other possible comparisons leading to the idea that the parts become smaller as objects of the same size are partitioned into a greater number of parts. The use of shapes on the flannel board illustrates these ideas quite well.

2. Prepare sets of cards with a fraction written on each card. A child may read the fraction and decide what fractional number must be added to equal "1". The cards can be made by the children and used for work with small groups or by pairs of children as a class activity.

3. Have ten members of the class stand in a line at the front of the room. Agree that each person is one tenth of the group and decide on the persons to be considered first and last in line. Let other children give directions such as:

   Will the first two tenths of you please tap your heads.
   Will the last three tenths please raise your hands.

4. Have the children look through old magazines to find and cut out pictures for making sets of 10 objects such as cars, cats, dogs, people, buildings, hats, or dresses. Mount these sets of 10 pictures on construction paper. Provide each child with a piece of string and ask the children to place the string around the subset that would show fractional parts of 10 such as seven tenths or five tenths. As the children work at their desks the same fractional parts may be shown on the flannel board. If time is limited, discs can be used instead of magazine pictures.

Content

FRACTIONAL NUMBERS

Equivalent Fractions

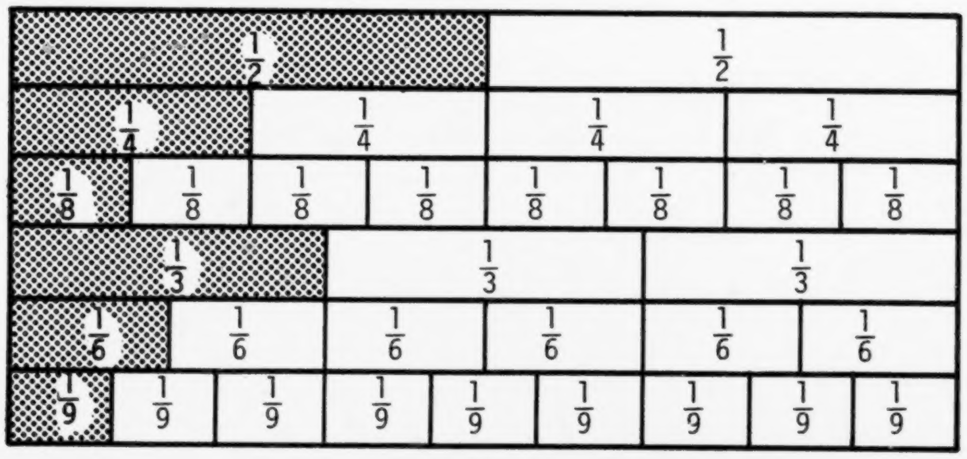
Objective

Given two equivalent fractions, the child can demonstrate that they are equal by using physical objects or diagrams.

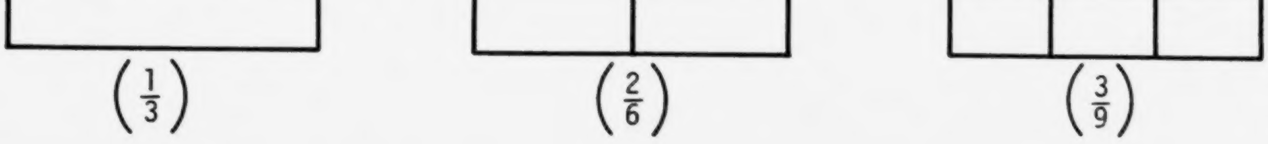
ACTIVITIES

1. Give each child a set of 8 objects. Ask how many objects are in one half of the set and four eighths of the set. (In each instance the answer is four.) Now ask if anyone can compare these four objects to the original set in still another way. (two fourths)

2. Duplicate a partitioned rectangle such as the following:


$\frac{1}{2}$ $\frac{1}{2}$
$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$
$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$
$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$
$\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$
$\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$ $\frac{1}{9}$

Your children can create their own models for these fractional numbers from colorful construction paper or you might provide them with tracing paper to copy a drawing similar to the one above which shows various parts. The students can then use their copy to measure unmarked rectangles such as those below and then make sentences about them.


$\left(\frac{1}{3}\right)$
$\left(\frac{2}{6}\right)$
$\left(\frac{3}{9}\right)$

Sample sentences: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$ $\frac{1}{3} = \frac{2}{6}$ $\frac{1}{3} = \frac{3}{9}$

Content

NATURAL NUMBERS

Roman Numerals
1 through XII

Objective

Given either Roman or Arabic numerals, the student can convert them to the other system of numeration.

ACTIVITIES

1. A clock face is probably the best teaching aid involving Roman numerals from 1 through XII. Three Roman numerals and their respective values are given below:

   I - 1
   V - 5
   X - 10

   By studying the Roman numerals used in the clock face sequence the students will detect the subtraction property peculiar to this system, i.e., whenever a symbol of lesser value is written before some other symbol, the lesser value is to be subtracted from the greater. Thus, IV represents four since 5 - 1 = 4.

2. Arrange as many as twelve students in a row and let each write the number of his position using Roman numerals.

NUMBERS E-1

Content

NATURAL NUMBERS

Cardinal Versus
Ordinal Use

Objective

Given a situation involving a number, the student can identify the use of the number as either cardinal or ordinal.

ACTIVITIES

1. Write a numeral on the chalkboard. Encourage students to make up a story involving both the cardinal and ordinal meaning of the number.

   Example: During the fifth period today, five students worked math exercises at the chalkboard. The fifth student worked his five exercises incorrectly.

   Now have the class identify each numeral as either the cardinal or ordinal use of the selected number. The imaginations of some students should produce rather clever stories.

NUMBERS E-2

Content

WHOLE NUMBERS

Skip Counting

Objective

The student is able to count by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's, and 10's in either increasing or decreasing order.

ACTIVITIES

1. Play a counting game. Here is one called Count By. This game may be played by a group of 3 students or by a class. One student is a counter, one a director and one or more students are judges. Three sets of cards are needed: a "start with" set each containing a numeral from 0 through 50, a "count by" set containing the numerals 2 through 9, and a set of cards with arrows to indicate the direction of the counting.

   The director shuffles the cards in each set and turns all sets face down. He appoints a counter. The counter draws from the first set of cards to determine the number from which he will begin counting. The counter then draws from the second set of cards to determine what he must count by, for example, 3's, 8's or 9's. From the third set of cards he draws a card which indicates whether he is to count forward or backward. If it is not possible to count in the direction indicated, then he must count in the other direction.

   The judges listen to the counter to determine if he makes an error. When the counter names up to 8 numbers correctly in the series, the director calls, "Stop," and the counter then becomes the director and appoints the next counter. If the counter fails to name all numbers in the series, the director would then appoint another counter.

Content

WHOLE NUMBERS

Place Value
Through One Million

Objectives

Given a numeral with several digits, the student can
1. read and
2. write
the place value of each digit.

ACTIVITIES

1. Write the numeral 7 on the chalkboard.

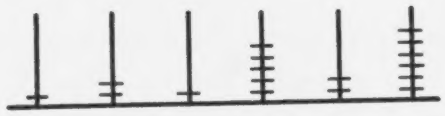
   Ask a student to read and interpret its meaning, for example, "7 means seven ones."

   Multiply 7 x 10 to get 70. Have students again read and supply a meaning, "70 means seven tens."

   Multiply 70 x 10 to get 700. Continue this pattern of multiplying each product by ten until you reach 700,000. If students do not know all the place values such as ten thousands or hundred thousands, have them write the word names.

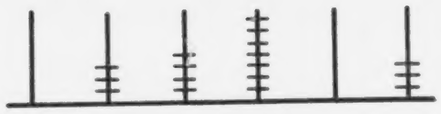
2. Using a six wire abacus or simply a sketch of an abacus, have the students name the place value of the wires, for example, ones, tens, hundreds, thousands, ten thousands, hundred thousands. Represent numbers on the abacus and have students write the numeral and say the number name. The students themselves might enjoy making up these numbers to challenge their classmates. Example:

   a.



(Answer: 121,526; one hundred twenty-one thousand, five hundred twenty-six)

   b.



(Answer: 34,703; thirty-four thousand, seven hundred three)

3. Encourage students to make up number puzzles and share them with the class, for example, What Is My Name?

   My name has six digits. My tens digit is 5. My ones digit is 4 less than my tens digit. My hundred thousands digit is the same as my thousands digit and is greater than three but less than five. My hundreds digit is 6 + 3, and my ten thousands digit is two less than my hundreds digit. Solution: My name is 474,951

   To find solutions to these puzzles more easily, students should make a place-value chart and enter the clues as they are given. In the above solution the tens digit would be written first, then the ones digit, the thousands digit, etc.

hundred thousands	ten thousands	one thousands	hundreds	tens	ones
4	7	4	9	5	1

Content

WHOLE NUMBERS

An Infinite Set

Objective

Given the set of whole numbers, the student can describe it as an unending set.

ACTIVITIES

1. The concept of an unending set can be developed through counting.

   Have volunteers from the class define sets of objects or numbers. Ask the children how many members are in each set. Any of these sets whose members can be listed and counted have a definite number of members.

   An example of such a set which the students might give is:

   A = {The odd numbers less than 12}

   This can also be listed as

   A = {1, 3, 5, 7, 9, 11}

   Here we have two different ways of naming the same set. How many members are in this set? (6)

   If, however, someone suggests a set like the following, C = {1, 3, 5, 7, 9, 11, ...}, then we know from the three dots that the set continues on indefinitely. Since we cannot list and count all the members of this set, we say the set of odd whole numbers is an unending or infinite set.

2. Challenge your students to give examples of unending or infinite sets. Notice that even though sets containing all the trees in the forests or all the grains of sand on the earth would have many members, they could be counted if there were enough time to do the counting. The students will probably conclude that all their examples of infinite sets will come from mathematics, either as sets of numbers or sets of points.

NUMBERS E-5

Content

WHOLE NUMBERS

Expanded Notation
Place Value
Through One Million

Objective

Given a six-digit numeral, the student can write it in expanded notation, for example:

532,748 = 500,000 + 30,000 + 2,000 + 700 + 40 + 8.

ACTIVITIES

1. To prepare students to accomplish this objective, have them write the simplest name for each of the following.

   a. 30,000 + 5,000 + 300 + 70 + 8 (35,378)

   b. 700,000 + 6,000 + 900 + 60 + 1 (706,961)

2. Have the students study and discuss the first exercise below and then complete the remaining ones.

   a. 2,175 = 2 thousands + 1 hundred + 7 tens + 5 ones
   = 2,000 + 100 + 70 + 5

   b. 73,429 = 7 ten thousands + 3 thousands + 4 hundreds + 2 tens + 9 ones
   = 70,000 + 3,000 + 400 + 20 + 9

   c. 284,567 = 2 hundred thousands + 8 ten thousands + 4 thousands + 5 hundreds + 6 tens + 7 ones
   = 200,000 + 80,000 + 4,000 + 500 + 60 + 7

NUMBERS E-6

Content

WHOLE NUMBERS

Order
Through One Million

Objective

Given a set of whole numbers, the student can arrange them from least to greatest and in reverse order.

ACTIVITIES

1. Write the numbers to be ordered on individual cards and then place the cards in jumbled order along the chalk tray. Have students compete to write the numbers as instructed from smallest to greatest or greatest to smallest. The first student to complete the ordering may come to the chalk tray and arrange the cards in correct sequence.

Content

WHOLE NUMBERS

Estimation
Rounding to
Tens
Hundreds
Thousands

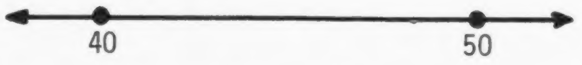
Objective

Given a number such as 4,631 the student can round it to the nearest ten, hundred, and thousand.

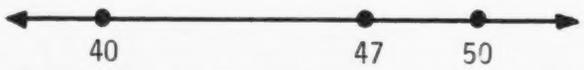
ACTIVITIES

1. Write the numeral 47. Ask your students to count by 10's from 0 to 100. Did they say the word "forty-seven" as they counted? (No) Why? (It is not ten or a multiple of ten.) Between what two tens does 47 belong? (40 and 50.)

   Show a segment of a number line between 40 and 50.

   Have a student show 47 on the number line.

   Is 47 nearer to 40 or to 50? (50) Since it is closer to 50, when we round 47 to the nearest ten, we round it to 50.

   Would 42 be rounded to 40 or 50? (40). What other whole numbers between 40 and 50 are rounded to 40? (41, 43, 44). While it is generally agreed that 45 is rounded to 50, and 55 is rounded to 60, this procedure for rounding when the final digit is 5 is not a hard-and-fast rule. Your students may enjoy searching for a different procedure to use in this special "in between" situation. Perform a similar activity to illustrate rounding a number like 734 to the nearest hundred.

2. The following activity might evolve from test scores or other available data. Have the children circle the correct answers.

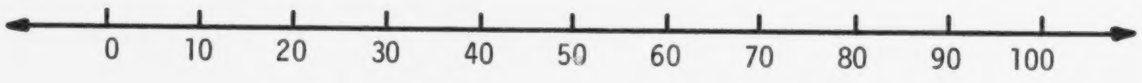
   a. Which of these numbers are written as 70 when they are rounded to the nearest ten?

   61, 63, [✓ 65], [✓ 67], [✓ 72], 75, 77.

   b. Which of these numbers are 200 when rounded to the nearest hundred?

   110, [✓ 176], [✓ 201], [✓ 224], 376, 300, 421.

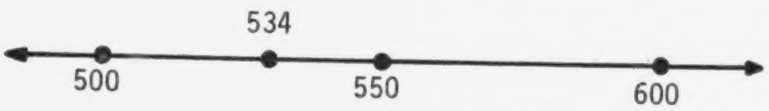
3. Have the students round these numbers to the nearest ten. They might use a ten number line to help them.

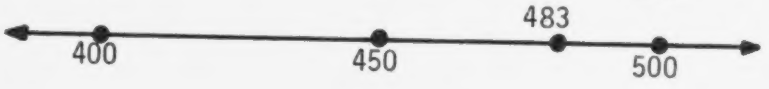
   a. 78 (80) b. 43 (40) c. 15 (20) d. 87 (90)

   This activity could be extended to include rounding numbers to the nearest hundred.

   e. 534 (500)

   f. 483 (500)

Content

WHOLE NUMBERS

Even
Odd

Objective

Given several whole numbers, the student can classify them as even or odd.

ACTIVITIES

1. Let's play a "remainders" game. Mentally divide each number by 2, forget the quotient and write down the remainder.

Number	Remainder
13	1
12	0
22	0
25	1
60	0

It appears that the only remainders which result from dividing by 2 are zero and one. Encourage students to suggest other numbers and divide by 2 until they feel sure that the only remainders possible are zero and one.

Have students look at the numbers with zero remainders. (12, 22, 60) What kind of numbers are these? (even) What kind of numbers have remainders of 1? (odd numbers: 13, 25)

Summarize or lead students to conclude that the even numbers are those with zero remainders when divided by 2. Their one's digit is 2, 4, 6, 8 or 0. Odd numbers have a remainder of one when divided by 2. Their final digit is 1, 3, 5, 7 or 9.

2. In the following set of numbers, draw a circle around each even number: {31, 17, 20, 32, 113, 210, 63, 87, 94, 10, 15, 72, 54, 8, 3, 1, 12, 19, 28, 37, 39, 50, 568, 763}

3. a. Name the even numbers between 3 and 17.

(4, 6, 8, 10, 12, 14, 16)

b. Name the odd numbers between 100 and 120.

(101, 103, 105, 107, 109, 111, 113, 115, 117, 119)

4. For the students who are very interested in the work of Activity 1, the rule of divisibility by two could be developed. This rule states that a number is divisible by two if the last digit of its numeral is divisible by two.

Content

WHOLE NUMBERS

Factors
Factorizations

Objective

Given a whole number such as 6, the student can write factorizations for it, for example:

6 = 6 x 1
6 = 2 x 3

ACTIVITIES

1. Have students find as many "multiplication names" or factorizations of 18 as possible.

   Review the terms, "factor" and "product," and their relationship:
   (factor) x (factor) = product.

   In finding factorizations of 18, students have been given a product and have found factors. From their answers have them make a list of the factors of 18. (1, 2, 3, 6, 9, 18)

2. Distribute 12 counters, grains of corn, or bottle caps to each of your students. Ask them to find all the ways they can arrange them with the same number of objects in each row. Instruct them to make a record of each arrangement. Six rows of two objects each would be recorded as "6 rows of 2" or "6 x 2". This activity should produce all the factors of 12.

   Now have your students list the factors of 12 on the chalk board. (Possible factorizations are: 1 x 12, 2 x 6, 3 x 4, 4 x 3, 6 x 2, 12 x 1. Thus the factors of 12 are 1, 2, 3, 4, 6, and 12.)

3. The following is an orderly procedure students can use to determine the factors of a given number. We shall illustrate it by finding the factorizations of 6.

   (a) 1 x 6 = 6
   (b) 2 x 3 = 6
   (c) 3 x 2 = 6
   (d) 4 x _ = 6
   (e) 5 x _ = 6
   (f) 6 x 1 = 6

   From the equations listed above, the factors of 6 are shown to be 1, 2, 3, and 6.

   Use this technique to determine the factors of other numbers such as 10, 18, 27, and 48.

Content

COMMON FACTORS

Greatest Common Factor
G. C. F.

Objectives

Given two whole numbers, the student can

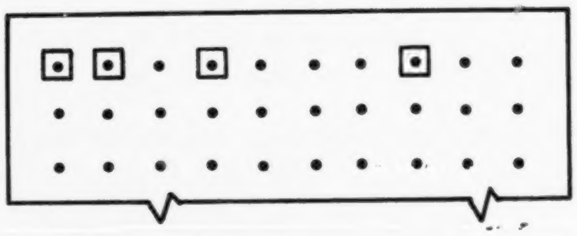
1. write their common factors and
2. name the greatest common factor.

ACTIVITIES

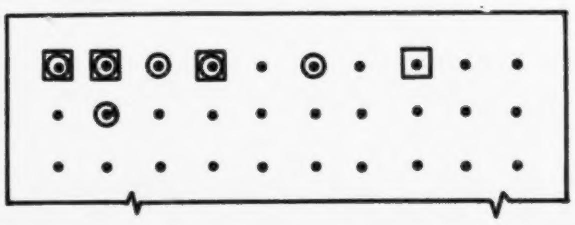
1. A hundreds board, a 10 by 10 geoboard or a grid will be helpful in illustrating to your students the meaning of common factors and greatest common factor.

   Have them record the factors of eight as they determine them through one of the procedures suggested in Numbers E-9.

   On a 10 by 10 geoboard this might be done with square washers.



   Then have them use round washers to indicate the factors of twelve.



   They will quickly see which factors are common to the two numbers. How? (The nail representing such a factor will have both a square and a round washer.) What are the common factors in this example? (1, 2, 4) Which one will be the greatest common factor? (The largest of these, 4)

2. Give your students exercises like those which follow. They might enjoy selecting the numbers themselves.

   a. List the factors of 6. (1, 2, 3, 6)

      List the factors of 16. (1, 2, 4, 8, 16)

      List the common factors of 6 and 16. (1, 2)

      Name the greatest common factor. (2)

   b. List the factors of 8. (1, 2, 4, 8)

      List the factors of 9. (1, 3, 9)

      List the common factors of 8 and 9. (1)

      Identify the G. C. F. (1)

Content

FRACTIONAL NUMBERS

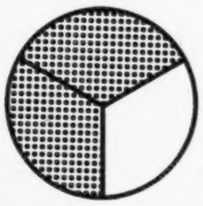
Numerator
Denominator

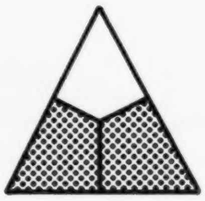
Objectives

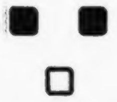
Given a fraction such as two thirds, the student can
1. identify and
2. name
the numerator and denominator.

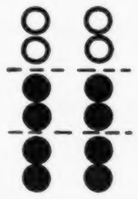
ACTIVITIES

1. Give your class a fraction and several models to illustrate that fractional number. In the sketches below, the shaded part of each figure or set represents $\frac{2}{3}$.









Have your students study these models and complete the following statements.

a. Each figure or set is divided into 3 equal parts.

b. 2 of the equal parts are shaded.

c. The denominator of a fraction tells the number of equal parts.

d. The numerator of a fraction tells the number of equal parts we are talking about.

2. Have the students complete the chart to demonstrate that they can distinguish between parts of a fraction.

Fraction	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{10}{20}$	$\frac{3}{4}$	$\frac{6}{8}$
Numerator	5	1	10	3	6
Denominator	9	3	20	4	8

Content

FRACTIONAL NUMBERS

- From Model to Name

Objectives

Given a model for a fractional number, the student can

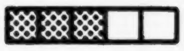
1. name and
2. write

a fraction for the number associated with the model.

ACTIVITIES

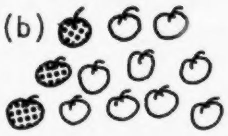
1. Provide the class with actual models or sketches such as those which follow. Below each object or set of objects have them write a fraction describing the shaded portion.

(a)



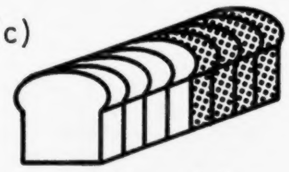
$\left(\frac{3}{5}\right)$

(b)



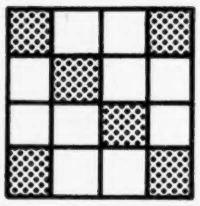
$\left(\frac{3}{12} \text{ or } \frac{1}{4}\right)$

(c)



$\left(\frac{4}{8} \text{ or } \frac{1}{2}\right)$

(d)



$\left(\frac{6}{16} \text{ or } \frac{3}{8}\right)$

2. Write fractions on the board. Have students read or write the word name. Example: You write "$\frac{3}{4}$" and students read or write *three fourths*.

3. It is essential that a student translate the word name of a fraction into the correct symbol.

   Read the following fractions and have students write down the proper symbol.

   a. one fourth $\left(\frac{1}{4}\right)$
   b. one half $\left(\frac{1}{2}\right)$
   c. five sevenths $\left(\frac{5}{7}\right)$
   d. two thirds $\left(\frac{2}{3}\right)$
   e. four fifths $\left(\frac{4}{5}\right)$
   f. nine twelfths $\left(\frac{9}{12}\right)$

Content

FRACTIONAL NUMBERS

Fractional Parts
Halves Through
Twelfths

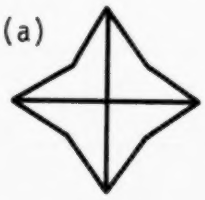
Objectives

Given a fractional number, the student can
1. construct and
2. identify
a model for it.

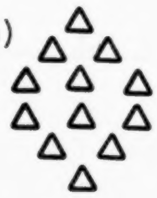
ACTIVITIES

1. Ask your students to prepare original sketches such as those shown below. Have them exchange sketches and identify a part such as three fourths of an object or set of objects.

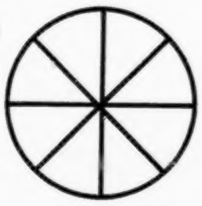
(a)



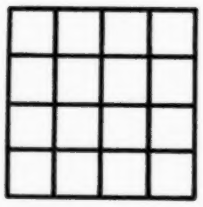
(b)



(c)



(d)

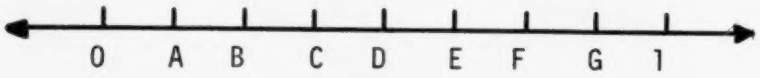


2. Divide the class into several groups of students and distribute to each group several pieces of construction paper and pairs of scissors. Challenge them to cut out and label as many fractional parts of the paper as they can representing halves, thirds, fourths, fifths, and so on up through twelfths.

   Students may fold or measure the paper. Let groups share experiences with the rest of the class. One third, for example, is difficult to fold so a group may be happy to tell their classmates how they determined one third of the paper by actual measurement. How would this group determine two thirds?

   A bulletin board displaying a whole sheet of construction paper and fractional parts made by the students would be an excellent summary of this activity.

3. Draw a number line on the chalkboard or overhead projector and label seven equally spaced points between 0 and 1



   You might then ask the class some of the following questions:

   (a) *Which point represents $\frac{1}{2}$?* (D)

   (b) *Which point represents $\frac{1}{4}$?* (B)

   (c) *What fractional number does point A represent?* ($\frac{1}{8}$)

   (d) *What fractional number does point F represents?* ($\frac{6}{8}$ or $\frac{3}{4}$)

Content

FRACTIONAL NUMBERS

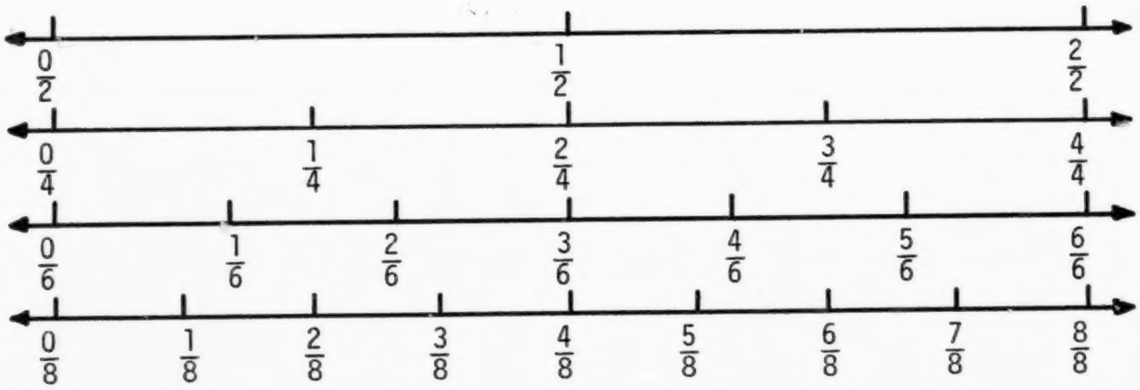
Equivalent Fractions

Objective

Given a fraction such as one half, the student can write a set of at least two equivalent fractions.

ACTIVITIES

1.

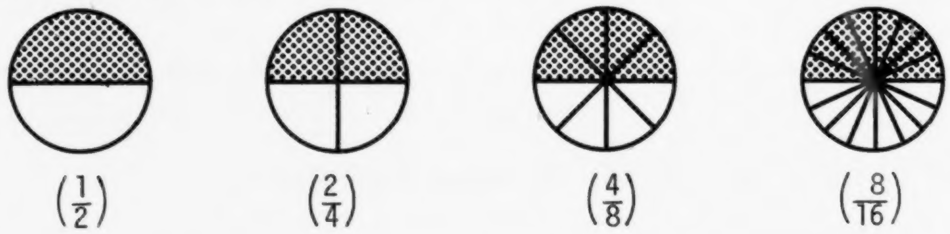


Display a series of number lines as above and have your students do the following:

a. Ask them what these fractions, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, have in common. (They all name the same fractional number.) We call these equivalent fractions and can write $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

b. From the number lines above have them find as many sets of equivalent fractions as they can. For example: $\left\{\frac{0}{2}, \frac{0}{4}, \frac{0}{6}, \frac{0}{8}\right\}$; $\left\{\frac{1}{4}, \frac{2}{8}\right\}$; $\left\{\frac{3}{4}, \frac{6}{8}\right\}$

2. On the chalk board, bulletin board, or overhead projector display models such as the following. Ask your students to write beneath each figure the fraction representing the shaded part of each circle and then have them do a through d below.


$\left(\frac{1}{2}\right)$
$\left(\frac{2}{4}\right)$
$\left(\frac{4}{8}\right)$
$\left(\frac{8}{16}\right)$

a. Compare the sizes of the circles. (They are all the same size)

b. Compare the sizes of the total shaded areas of the circles. (They are all the same size)

c. Doesn't this indicate that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$? Since $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$ and $\frac{8}{16}$ name the same number, they are called equivalent fractions.

d. Suppose we draw another circle the same size as those above and equally divide it into 32 parts. How many of the equal parts will we have to shade so that $\frac{1}{2}$ of the circle would be shaded? (16) Is $\frac{16}{32}$ equivalent to $\frac{1}{2}$? $\frac{2}{4}$? $\frac{8}{16}$? (Yes, in each instance)

Content

FRACTIONAL NUMBERS

Equivalent Fractions
Simplest Form

Objective

The student can determine the simplest fraction for naming a fractional number.

ACTIVITIES

1. Display models like those used in the activities for Numbers E-14.

   a. From models one can see that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{8}{16}$ are equivalent fractions. Which name is the simplest? (one half because it involves smaller numbers)

   b. Models have also shown that $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$. Which is the simplest name? $\left(\frac{2}{3}\right)$

   c. Through questioning try to get students to see that $\frac{1}{2}$ and $\frac{2}{3}$ are the simplest name because the numerator and denominator in each case have no common factor except 1.

2. Give the students sets of equivalent fractions. Have them find the fraction in each set which has the simplest name.

   a. $\left\{\frac{4}{6}, \frac{2}{3}, \frac{12}{18}, \frac{14}{21}, \frac{20}{30}, \frac{10}{15}\right\}$ $\left(\frac{2}{3}\right)$

   b. $\left\{\frac{1}{5}, \frac{20}{100}, \frac{2}{10}, \frac{5}{25}, \frac{10}{50}, \frac{6}{30}\right\}$ $\left(\frac{1}{5}\right)$

   c. $\left\{\frac{4}{2}, \frac{8}{4}, \frac{2}{1}, \frac{10}{5}, \frac{6}{3}, \frac{20}{10}\right\}$ $\left(\frac{2}{1}\right)$

3. A set of dominoes may be used effectively to help teach fractions. Hold up a domino in vertical position putting the larger number of dots on the bottom. Explain that the domino represents a fraction, the upper dots representing the numerator and the lower dots representing the denominator. Have the students tell you whether or not the fraction represented is in simplest form; if it is not, ask them to tell you the simplest name.

Content

FRACTIONAL NUMBERS

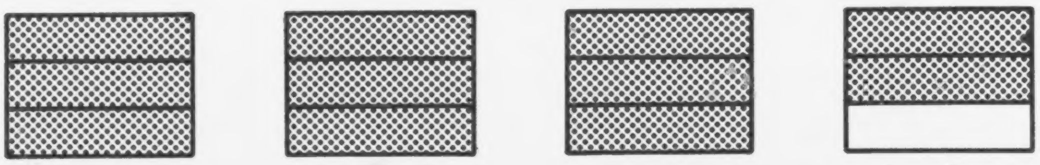
Mixed Numerals
From Model to Numeral

Objective

Given a model for a number such as $3\frac{2}{3}$, the pupil can write the fraction $\frac{11}{3}$ or the mixed numeral $3\frac{2}{3}$.

ACTIVITIES

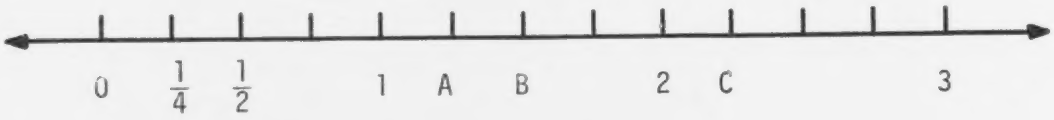
1. Present this model to your class and ask them to name the shaded area in at least two different ways.



$$\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right) + \left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right) + \left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right) + \left(\frac{1}{3}+\frac{1}{3}\right) = \frac{11}{3}$$

$$1 + 1 + 1 + \frac{2}{3} = 3\frac{2}{3}$$

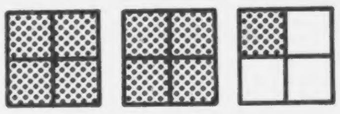
2. Have your students discuss that there are many names for the same number. Ask them to name points A, B, and C on the number line below in at least two ways. Use a mixed numeral for one of the names.



A $(1\frac{1}{4}, \frac{5}{4})$ B $(1\frac{1}{2}, 1\frac{2}{4}, \frac{3}{2}, \frac{6}{4})$ C $(2\frac{1}{4}, \frac{9}{4})$

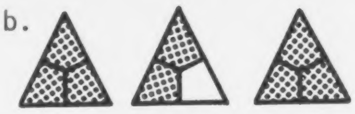
3. Use diagrams of shapes or number lines for additional experience in working with mixed numerals. Some children might enjoy creating their own diagrams to be worked by others. In the examples which follow, have the class give more than one name for the shaded areas or lettered points on the number lines.

a.



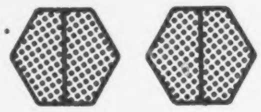
$(\frac{9}{4}, 2\frac{1}{4})$

b.



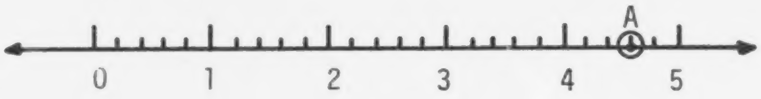
$(\frac{8}{3}, 2\frac{2}{3})$

c.



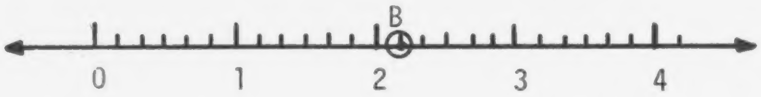
$(\frac{4}{2}, 2)$

d.



A $(\frac{23}{5}, 4\frac{3}{5})$

e.



B $(\frac{13}{6}, 2\frac{1}{6})$

Content

FRACTIONAL NUMBERS

Mixed Numerals
From Numeral to Model

Objective

Given a mixed numeral such as $1\frac{1}{2}$, the student can create a model to illustrate that $1\frac{1}{2} = 1 + \frac{1}{2}$.

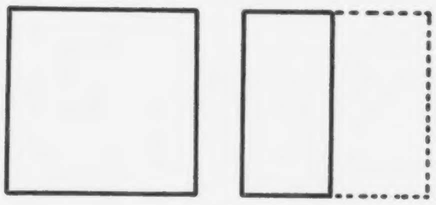
ACTIVITIES

1. Provide the class with a variety of materials such as bottle caps, straws, dried beans, fraction kits and number lines. From this assortment or their own imaginations expressed through sketches, challenge your children to create models to represent various mixed numerals you wish to give them.

   Encourage them to draw their own illustrations for numbers you have selected.

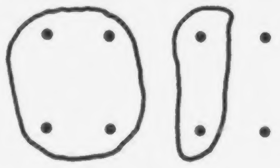
   For example, $1\frac{1}{2}$ could be illustrated by any of the following sketches.

(a)



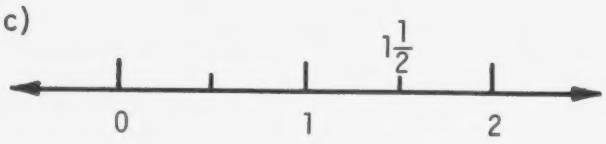
$(1 + \frac{1}{2}) = 1\frac{1}{2}$

(b)



$(1 + \frac{1}{2}) = 1\frac{1}{2}$

(c)



Content

FRACTIONAL NUMBERS

Order
<, =, >

Objectives

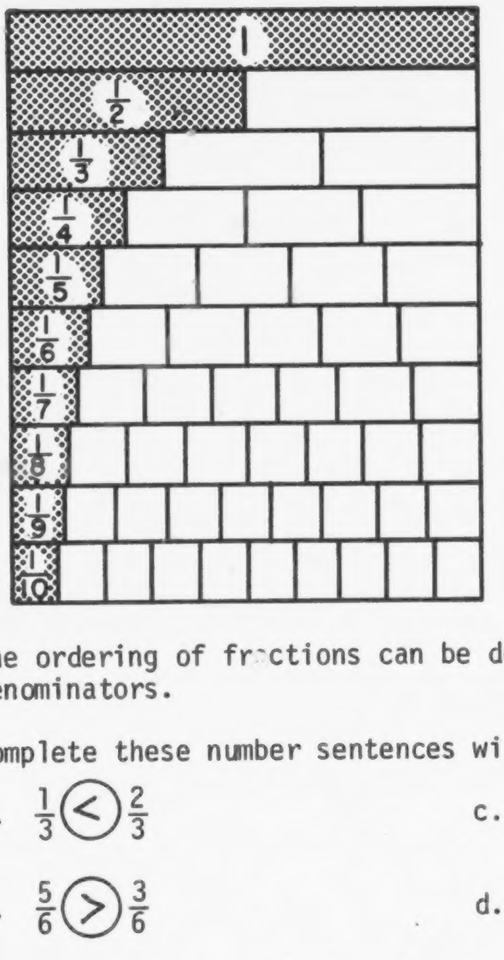
Given two fractional numbers such as $\frac{1}{2}$ and $\frac{5}{6}$, the student can

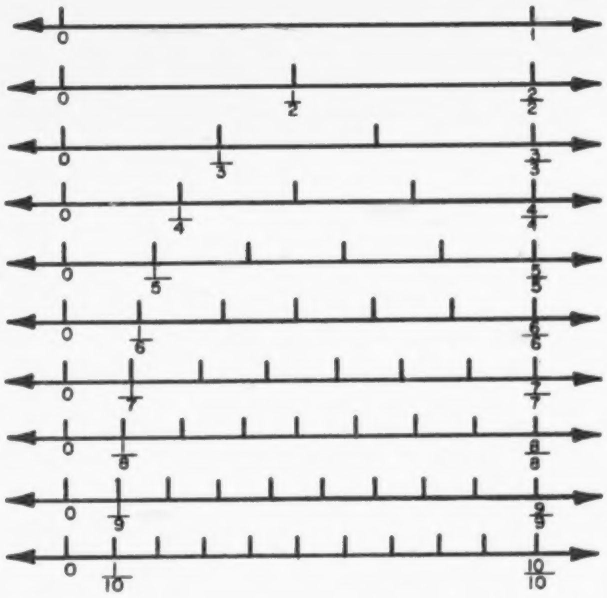
1. rename them as fractions with common denominators and
2. write one of these symbols <, =, or > to express the order relationship between them, for example:

$$\frac{3}{6} < \frac{5}{6}$$

ACTIVITIES

1. Allow students to view a fraction chart or a series of number lines to perform the exercises in this activity.





The ordering of fractions can be done quickly and conveniently if the fractions have common denominators.

Complete these number sentences with <, =, or >

a. $\frac{1}{3} < \frac{2}{3}$

b. $\frac{5}{6} > \frac{3}{6}$

c. $\frac{3}{8} < \frac{4}{8}$

d. $\frac{4}{5} > \frac{3}{5}$

e. $\frac{1}{4} < \frac{3}{4}$

f. $\frac{2}{3} < \frac{5}{6}$

g. Mary's mother served apple pie for dinner. She cut the pie into six slices. Mary ate one slice, but her teenage brother ate two slices. How much of the pie did Mary eat? (one sixth). How much of the pie did her brother eat? (two sixths). Who ate the most pie, Mary or her brother? (her brother since $\frac{2}{6} > \frac{1}{6}$)

h. Complete the sentence: If the denominators are the same, the greater the numerator, the greater the fractional number. See if your students can make a similar statement about fractions whose numerators are the same.

Content

NATURAL NUMBERS

Roman Numerals
I, V, X, L, C, D, M

Objective

Given Roman and Arabic numerals, the student can convert them from Roman to Arabic and from Arabic to Roman.

ACTIVITIES

1. Discuss the meaning of the symbols I, V, X, L and C.

   Have a student look up the word cent in the dictionary. What was its meaning in the Latin language? This dictionary exercise should reveal why there are 100 cents in a dollar. Your students will now better understand how the Roman numeral C came to represent 100.

2. Have a committee of students make a clock face that has Roman numerals. Perhaps students have seen such a clock face, and would like to tell the class about it. Roman numerals are also frequently used on the faces of sundials.

3. The volumes of some sets of encyclopedias or other reference books are numbered with Roman numerals. If possible bring such a set into the classroom. Let a student select at random a volume, read the Roman numeral to the class and challenge them to find the corresponding Arabic numeral.

4. Let children represent in Roman numerals many numbers from their experience such as the amount of their lunch money, their age, the number of brothers and sisters they have, the number of students in the class, the number of blocks or miles they live from school, etc.

5. Write the letter of the Arabic numeral in the second column that corresponds to the Roman numeral in the first column. Note that the second column has more entries than the first.

First Column	Second Column
f 1. II	a. 6
g 2. IV	b. 20
a 3. VI	c. 40
c 4. XL	d. 75
b 5. XX	e. 33
	f. 2
	g. 4

6. Extend this topic to include the Roman Numerals D and M for those students who are interested. Have them find examples where these larger numbers are used. (A few such examples are to be found in the publication date of some books, the production year of films, and cornerstones of buildings.)

Content

WHOLE NUMBERS

Counting
Through One Billion

Objective

Given a whole number, the student can write several consecutive numbers which precede and follow it.

ACTIVITIES

Have your students:

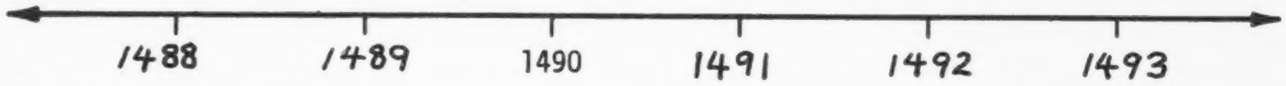
1. Record the numbers which come before and after the one given in each exercise below.

a. 23 24 25 26 27 28 29

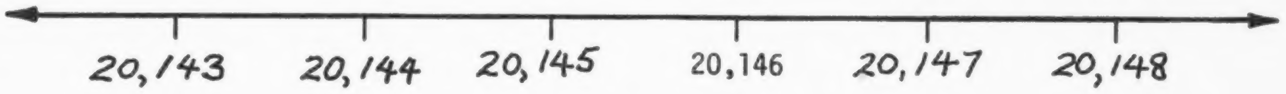
b. 996 997 998 999 1000 1001 1002

2. Copy and complete the number lines below:

a.


1488
1489
1490
1491
1492
1493

b.


20,143
20,144
20,145
20,146
20,147
20,148

3. Name the number which is 100,000 greater than each of the following:

a. 36,492 (136,492)

b. 910,459 (1,010,459)

Then have them name the number which is 1,000,000 less than:

c. 1,000,497 (497)

d. 697,453,192 (696,453,192)

Content

WHOLE NUMBERS

Place Value
Nine-Digit Numerals

Objectives

Given a nine-digit base ten numeral, the student can
1. identify and
2. name
the place value of each digit.

ACTIVITIES

1. Develop with the students a place-value chart for nine-digit numerals.

Millions			Thousands					
Hundred	Ten	One	Hundred	Ten	One	Hundreds	Tens	Ones
				7	4	3	6	1
6	8	9	5	2	4	0	3	1

Write several numerals on the chalkboard and have students write them in the proper position in the place-value chart.

2. Have them complete the following

   a. Consider the numeral 74,361

   The 4 names the number of (thousands)

   The 7 names the number of (ten thousands)

   b. Consider the numeral 689,524,031

   The 8 names the number of (ten millions)

   The 2 names the number of (ten thousands)

   The 0 names the number of (hundreds)

3. Give your students information such as the following from which they may "build" a number.

   1 hundred, 3 thousands, 4 ten thousands, 0 tens, 5 hundred thousands, 9 ones. (543,109)

Content

WHOLE NUMBERS

Place Value
Period
 Ones
 Thousands
 Millions

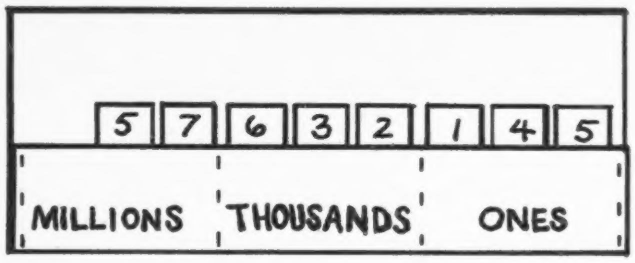
Objectives

Given a nine-digit numeral, the student can
1. identify and
2. name

the period value for each group of 3 digits.

ACTIVITIES

1. Write 57632145 on the chalkboard or overhead projector. Do not put commas in the numeral. Ask the students to read it. They will probably have difficulty and may suggest that commas be used for easier reading. This will be a natural way to discuss the periods contained in the numeral.



To illustrate this you might wish to construct a place-value chart which can be easily made by folding and stapling a large sheet of tagboard as shown in the illustration on the left. Replaceable numeral cards can then be placed in the pockets to represent various numbers.

2. Have your students complete the following statements.

   a. Look at the numeral 234,567,891. 234 is in the millions period, 567 is in the thousands period and (891) is in the ones period.

   b. In the numeral 13,031,567 (13) is in the millions period and (567) is in the ones period.

   c. In the numeral 6,000,000 (0) is in the ones period, (0) is in the thousands period and (6) is in the millions period.

   Conclusion: As we move from right to left in examining the digits of a numeral, the first three digits represent ones, the next three digits represent thousands, and the next three digits represent millions.

*3. Expanded notation can be used to illustrate the periods of a numeral. For example, 234,567,890 means 234 millions plus 567 thousands plus 890 ones. This can be written as (234 x 1,000,000) + (567 x 1,000) + (890 x 1). Give your students examples such as the following to complete.

   a. 32,115,692 means 32 millions + 115 thousands + 692 or
   (32 x 1,000,000) + (115 x 1,000) + (692 x 1)

   b. 567,439,116 = (567 x 1,000,000) + (439 x 1,000) + (116 x 1)

Content

WHOLE NUMBERS

Description of Large Numbers

Objective

Given a number greater than 1000, the student can explain his notion of the quantity involved. For example: "A million dollars is having a thousand dollars in each of a thousand banks."

ACTIVITIES

1. To accomplish this most effectively, challenge your students to develop or create their own explanations of the meaning of bigness in numbers. Time and effort spent by them in preparing original interpretations will likely make more of an impression than reading about large numbers in books. From the interest generated by the explanations of their classmates, some students may then wish to find references to large numbers in books.

2. Ask questions whose answers involve estimation. This will likely generate a great deal of interest among your students. Be sure to encourage them to make guesses before any measurements or computations are made. Some sample questions follow.

   a. *If you were to begin counting from one to one million at the rate of one number per second and not take a break until you had reached your goal, how long would it take? (Don't accept one million seconds from your students. Ask for an answer in terms of hours, days, weeks, months or years. Notice how wide a variation there is in the answers. Use this difference in estimations to provide a motivated setting for computing the answer.)*

   b. *How tall would a stack of one million nickels be? Would it be higher than your school building, or Cape Hatteras Lighthouse, or Grandfather Mountain? How much money is one million nickels?*

Content

WHOLE NUMBERS

Expanded Notation
Nine-Digit Numerals
Place Value

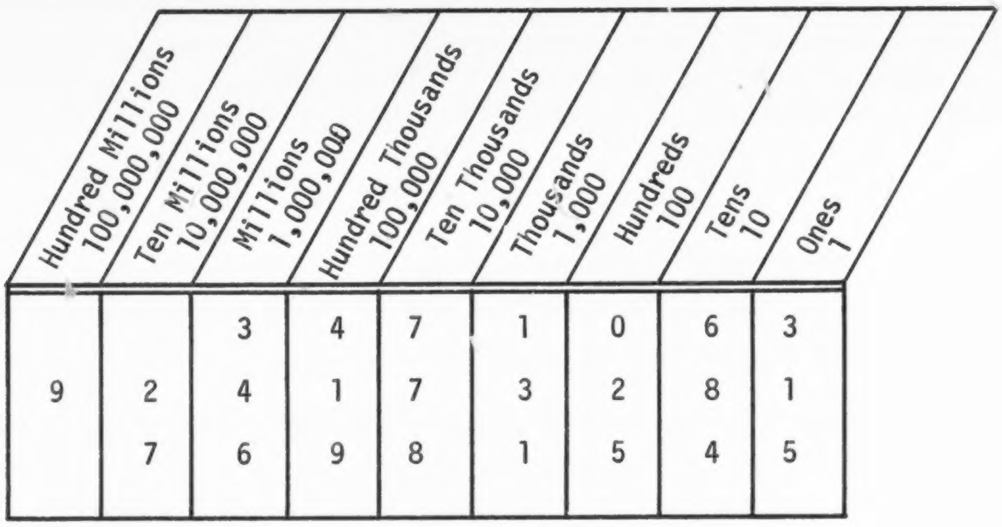
Objective

Given a numeral such as 243,765,108 the student can write it in expanded notation. For example:

243,765 = (2 x 1,000,000,000) + (4 x 10,000,000) + (3 x 1,000,000) + (7 x 100,000) + (6 x 10,000) + (5 x 1,000) + (1 x 100) + (0 x 10) + (8 x 1).

ACTIVITIES

1. a. With the students' help make another place-value chart including both word names and numeral names for the positional value of each digit.



Hundred Millions 100,000,000	Ten Millions 10,000,000	Millions 1,000,000	Hundred Thousands 100,000	Ten Thousands 10,000	Thousands 1,000	Hundreds 100	Tens 10	Ones 1
		3	4	7	1	0	6	3
9	2	4	1	7	3	2	8	1
	7	6	9	8	1	5	4	5

b. After writing several numerals on the chart as shown above, have students write expanded notation for them as illustrated in the first example below.

3,471,063 = (3 x 1,000,000) + (4 x 100,000) + (7 x 10,000) + (1 x 1,000) + (0 x 100) + (6 x 10) + (3 x 1)

924,173,281 = (9 x 100,000,000) + (2 x 10,000,000) + (4 x 1,000,000) + (1 x 100,000) + (7 x 10,000) + (3 x 1,000) + (2 x 100) + (8 x 10) + (1 x 1)

76,981,545 = (7 x 10,000,000) + (6 x 1,000,000) + (9 x 100,000) + (8 x 10,000) + (1 x 1,000) + (5 x 100) + (4 x 10) + (5 x 1)

2. Ask students to read large numbers to the class and have the class write these in expanded notation. Require students to write numerals on the chart only if they cannot write the expanded numeral without this aid.

Content

WHOLE NUMBERS

Order
Through One Billion

Objective

Given a set of whole numbers, the student can write them in order from least to greatest and vice versa.

ACTIVITIES

1. The books in the school library have numerals printed on their spines. Perhaps your students have studied the Dewey Decimal System and know that many of the books on library shelves are placed in numerical order. Books with the following numerals were placed on the "return desk" at the library: 386, 432, 819, 999, 004, 163, 577, 561, 704, 063, 892, and 711. Ask the students to help the librarian by arranging the books in numerical order beginning with the smallest. (004, 063, 163, 386, 432, 561, 577, 704, 711, 819, 892, 999)

2. Have them arrange these test scores from highest to lowest: 58, 89, 76, 96, 83, 69, 85, 74

   (96, 89, 85, 83, 76, 74, 69, 58)

3. Direct your students to write the numerals in each exercise beginning with the smallest or largest as indicated:

   a. 7408, 3602, 3901, 4258, 3048, 6607 - begin with the smallest

      (3048, 3602, 3901, 4258, 6607, 7408)

   b. 239,678; 677,120; 8,713,465; 920,067,214; 147,603,982; 34,900 - begin with the largest

      (920,067,214; 147,603,982; 8,713,465; 677,120; 239,678; 34,900)

Content

WHOLE NUMBERS

Word Names
Through One Billion

Objective

Given a numeral such as 4,678,047 the student can write the words: four million, six hundred seventy-eight thousand, forty-seven, and vice versa.

ACTIVITIES

1. Ask students to write four or five numerals each with nine digits or less on a sheet of paper. Then have them write the words they would say in reading the numerals.
2. Have students locate information containing very large numbers and read these statements to the class. Facts about national and state budgets, and space flight are excellent sources of information.
3. Not all numbers are read as this objective states. Telephone numbers, automobile license plate numbers, social security numbers, and serial numbers are some exceptions. The telephone number 467-8047 is read four-six-seven-eight-zero-four-seven. Challenge the student to think of these and other exceptions.

Content

WHOLE NUMBERS

Word Names
Through One Billion

Objective

Given a word phrase naming a number, the student can write the word name and the Arabic or standard numeral.

ACTIVITIES

1. Have the students select teams of three persons each. One names a number; another then records it with numerals on a flannel, magnetic, or chalk board; and the third writes the number words on the chalk board or overhead projector.

   Pupils who have difficulty should be encouraged to use an abacus, a counting frame, place-value chart, or some other concrete device plus word cards (ones, tens, hundreds, thousands) or a printed word list until they experience some measure of success.

Content

WHOLE NUMBERS

Estimation
Rounding
Through Hundred
Thousands

Objective

Given a number such as 246,725 the student can round it to the nearest ten, hundred, thousand, etc.

ACTIVITIES

1. Present the following examples to your students.

   The number 246,725 rounded to the nearest ten is 246,730 ;

   The number 246,725 rounded to the nearest hundred is 246,700 ;

   The number 246,725 rounded to the nearest thousand is 247,000 ;

   When doing problems of this type, your students should:

   a. Always go back to the original number to do the rounding, and

   b. In the case of a final 5 or 50 or 500, be aware that the number is usually increased as shown in rounding 246,725 to 246,730. Your students may enjoy searching for a different rule to use in these special situations.

2. Have your students collect various bits of information such as the following:

   The lowest temperature reading in North Carolina last night. __________

   The highest temperature reading in North Carolina yesterday. __________

   The number of students enrolled in this school. __________

   The latest population report available for the State __________, County __________, City __________.

   The dates covering the settlement of the thirteen colonies. __________ to __________

   The elevation of the highest peak in North Carolina. __________

   The length of the coastline of North Carolina. __________

   Include other statistical information which might interest the children. As these figures are discussed have the students round them to the nearest ten, hundred, and thousand, where possible. Have them suggest when and why the rounding of numerals is expedient.

3. Ask the class to list some instances when they would use large numbers having four digits or more. Ask them to record their examples and then round them to the nearest ten, hundred, and thousand.

4. Before they actually do the following computation, have the students estimate whether the sum of the given addends is more or less than the sum of the given number, for example:

   Is the sum (1572 + 1784) more than or less than 3000? (Since 1600 + 1800 = 3400, the answer is *more than*.)

   Encourage your students to make up several of these to challenge their classmates.

Content

WHOLE NUMBERS

Prime
Composite

Objective

Given a whole number from 1 through 100, the student can classify the number as being prime when it has exactly two factors or composite when it has more than two factors.

ACTIVITIES

1. A sheet of graph paper containing a 10 by 10 array of squares is an effective way to have students determine all the prime numbers in the set of whole numbers from 1 through 100. Direct your students to write the Arabic numerals for these numbers as illustrated by the part of the grid shown below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

After defining prime and composite numbers and removing 1 from the set since it is a special number fitting neither definition, have the students circle the next numeral, 2, and then cross out any others that have two as a factor.

	②	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~

Now instruct them to circle the next remaining numeral which is 3 and cross out all multiples of three which remain.

	②	③		5		7		~~9~~	
11		13		~~15~~		17		19	

By continuing this process your students will have identified all the prime numbers in this set of whole numbers. This procedure is often referred to as the Sieve of Eratosthenes. Some students may enjoy finding out why it was given this name.

Have your students compare their charts to see if they found the same numbers.

(Continued on next page)

2. A second activity to use in determining whether a number is prime or composite involves the listing of factors.

Number	Factors
1	1
2	1,2
3	1,3
4	1,2,4
5	1,5
6	1,2,3,6
7	1,7

A prime number has exactly two factors and a composite number has more than two factors. With these definitions your students can quickly classify numbers by developing the list at the left for themselves. Note that the number 1 falls under neither definition.

As a follow-up activity have the students classify several numbers as prime or composite by listing their factors, for example:

a. 7 (1, 7; prime)
b. 10 (1, 2, 5, 10; composite)
c. 9 (1, 3, 9; composite)
d. 11 (1, 11; prime)

NUMBERS F-11

Content

WHOLE NUMBERS

Even and Odd
Prime

Objectives

Given a whole number between three and one hundred, the student can

1. classify it as even or odd, and
2. write it as the sum of prime numbers.

ACTIVITIES

1. Have students review ways of determining whether a number is even or odd. (See Numbers E-8.) Then have them make a list of all prime numbers less than one hundred. (See Numbers F-10.)

   Have them begin with the number four and list each whole number as even or odd, and name it as the sum of two primes if possible. If not, try to use no more than three primes.

Even	Odd
4 = 2 + 2	5 = 2 + 3
6 = 3 + 3	7 = 2 + 5

   Ask the students to make a final tabulation of their results to see if they used the fewest number of primes for each number and to find alternate names when they exist.

Content

WHOLE NUMBERS

Prime Factorization

Objective

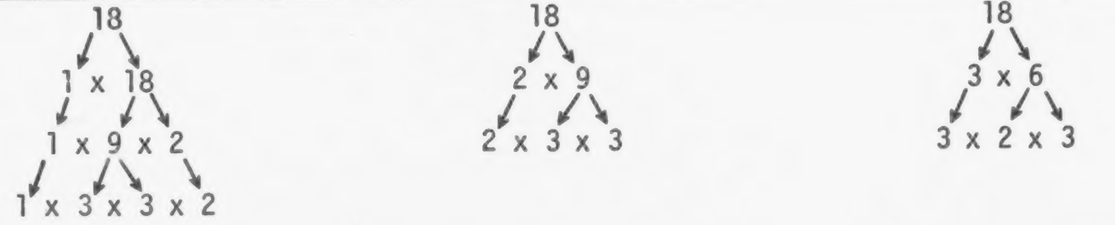
Given a composite number, the student can write it as the product of prime numbers.

ACTIVITIES

1. Ask your students to find several factorizations of 18. They should get the following:

   1 x 18 2 x 9 3 x 6

   Now ask the students to find the prime factors of each factorization and arrange them in order from smallest to largest. The use of factor trees will simplify this work.



18 = 1 x 3 x 3 x 2	18 = 2 x 3 x 3	18 = 3 x 2 x 2
*18 = 1 x 2 x 3 x 3	18 = 2 x 3 x 3	18 = 2 x 3 x 3

*Since the factor 1 does not affect the product, this prime factorization may be written as 18 = 2 x 3 x 3

Although different factors were used in the first step of each example, your students will see that the prime factors are all the same. Have them try additional examples to see if other composite numbers have only one prime factorization.

2. Have the students write prime factorizations for each of the following composite numbers. They may wish to use factor trees.

   a. 6 = (2 x 3)
   b. 8 = (2 x 2 x 2)
   c. 25 = (5 x 5)
   d. 45 = (3 x 3 x 5)

Content

WHOLE NUMBERS

Greatest Common Factor
G. C. F.

Objective

Given two whole numbers, the student can use prime factorization to determine their greatest common factor.

ACTIVITIES

1. Have the students discuss and give examples to show they understand that *factor times factor equals product.* Have them record the different multiplication facts which name the same product by asking such questions as, "What are all the factorizations of 18?" Let them decide whether the factors of a given number constitute a set. Have them name and record these numbers in the following way:

   The factors of 18 are 1, 2, 3, 6, 9, and 18.

   The factors of 12 are 1, 2, 3, 4, 6, and 12.

   Compare the two sets to identify common factors. (1, 2, 3, 6) Ask the students what name they might give to the largest of these. (the greatest common factor) In the example above what is the greatest common factor? (6)

Content

WHOLE NUMBERS

Exponential Notation
of 10

Objectives

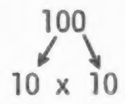
Given a number such as 1,000 the student can

1. write it in exponential form and
2. identify the base and exponent.

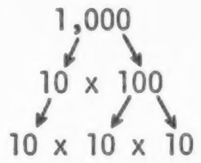
ACTIVITIES

1. Have your students develop the following factor trees using only 10's and powers of 10.

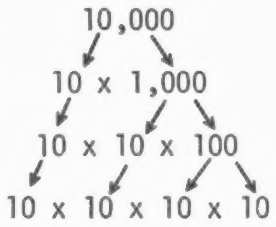
a.



b.



c.



2. The above information may be summarized as follows:

100 = 10 x 10 1,000 = 10 x 10 x 10 10,000 = 10 x 10 x 10 x 10

Have your students study this summary and try, without the use of factor trees, to write the factorizations of 10 for the three examples which follow.

100,000 = (10 x 10 x 10 x 10 x 10)

1,000,000 = (10 x 10 x 10 x 10 x 10 x 10)

10,000,000 = (10 x 10 x 10 x 10 x 10 x 10 x 10)

3. Suggest that there is a shorter way to write these long lists of factors. Write the following for the class to see.

$100 = 10^2$ $1000 = 10^3$ $10{,}000 = 10^4$

Ask students to study the pattern and try to write the following numbers using this mathematical shorthand.

$100{,}000 = (10^5)$ $1{,}000{,}000 = (10^6)$

Explain that in the above examples 10 is called the base and the 2, 3, 4, etc., are called exponents.

Ask the class what these exponents represent. Their answers will vary. Using the following example, $100 = 10^2 = 10 \times 10$, you will probably find your students making these two observations:

a. The exponent, 2, indicates how many times the base is used as a factor, 10 x 10, and

b. The exponent, 2, shows how many zeroes are in the standard Arabic numeral, 100.

Content

WHOLE NUMBERS

Least Common Multiple
L. C. M.

Objective

Given two whole numbers, the student can use prime factorization to determine their least common multiple.

ACTIVITIES

1. On the chalk board or overhead projector have students list the first few multiples of each of the following:

   Multiples of 2: (0, 2, 4, 6, 8, 10, 12, ...)

   Multiples of 3: (0, 3, 6, 9, 12, 15, 18, ...)

   Multiples of 4: (0, 4, 8, 12, 16, 20, ...)

   Multiples of 5: (0, 5, 10, 15, 20, 25, ...)

   Multiples of 6: (0, 6, 12, 18, 24, 30, ...)

   It appears that zero is a multiple of every number. Why? (zero times any number equals zero. Since this is so, zero will be disregarded in this discussion.)

   Are there some multiples common to the multiples of 2 and 3? (Yes, 6, 12, and 18) Can you think of other common multiples of 2 and 3? (24, 30, 36, ...)

   Find the multiples common to both 4 and 6. (12, 24, 36, ...) How many are there? (An unlimited or infinite number) What is the least common multiple of 4 and 6? (12)

2. Have the students use prime factorization to find the L. C. M. of two numbers such as 12 and 18.

   12 = 2 x 2 x 3   18 = 2 x 3 x 3

   Since "2" occurs twice as a factor of 12, and "3" is present two times in the prime factorization of 18, the least common multiple of these two numbers will be: (2 x 2 x 3 x 3) or 36.

Content

FRACTIONAL NUMBERS

Numerator
Denominator

Objectives

Given a fraction such as $\frac{1}{2}$, the student can

1. identify the denominator,
2. state that it indicates the number of parts into which a region, a set of objects, or a number line segment has been divided;
3. name the numerator, and
4. state that it tells how many of these parts are being considered.

ACTIVITIES

1. a. Review with your students that $\frac{2}{3}$ is a fraction, the numerator is 2 and the denominator 3.

   b. After reviewing this, write $\frac{\square}{\triangle}$ to represent a fraction and ask "What is the denominator?" (△); "What is the numerator?" (□)

   It is important that the students be able to answer such questions as:

   "What numbers can replace □ ? (All whole numbers.)

   "What numbers can replace △ ? (All whole numbers except zero)

   Even though students have been exposed to this previously, review with them the meaning of these terms by asking such questions as,

   "What does the numerator tell if we are referring to $\frac{3}{4}$ of a set? ... to $\frac{3}{4}$ of an object?"

   "What does the denominator tell if we are referring to $\frac{2}{3}$ of a set? ... to $\frac{2}{3}$ of an object?"

2. Have your students write five fractions as described below (Answers will vary in a and b below.)

Numerator	Denominator	Possible Answers
a. any whole number	9	(Sample answer: $\frac{0}{9}, \frac{1}{9}, \frac{3}{9}, \frac{19}{9}, \frac{27}{9}$)
b. 19	different whole numbers	(Sample answer: $\frac{19}{1}, \frac{19}{2}, \frac{19}{19}, \frac{19}{20}, \frac{19}{100}$; $\frac{19}{0}$ is not acceptable)
c. 5, 6, 7, 8, 9	6 times the numerator	($\frac{5}{30}, \frac{6}{36}, \frac{7}{42}, \frac{8}{48}, \frac{9}{54}$)
d. one half the denominator	32, 88, 12, 18, 20	($\frac{16}{32}$ $\frac{44}{88}$ $\frac{6}{12}$ $\frac{9}{18}$ $\frac{10}{20}$)

Content

FRACTIONAL NUMBERS

Whole Numbers
as Fractions

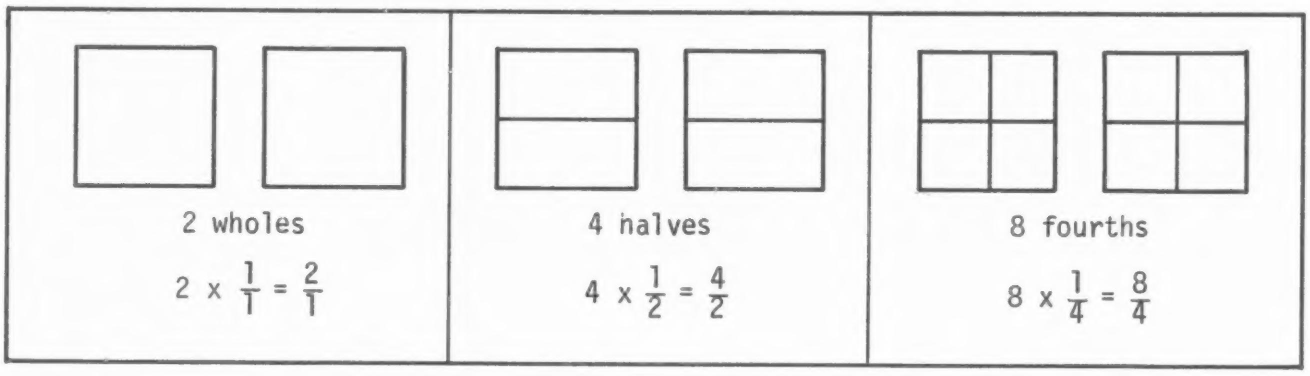
Objective

Given a whole number such as 2, the student can write it as a fraction $\frac{2}{1}$ or $\frac{4}{2}$, and vice versa.

ACTIVITIES

1. Drawings of figures which have been divided into different numbers of parts will demonstrate the naming of a whole number as a fraction.

   Have your students draw several pairs of figures and divide each set of two into a different number of parts such as halves, fourths, etc.


2 wholes
$2 \times \frac{1}{1} = \frac{2}{1}$
4 halves
$4 \times \frac{1}{2} = \frac{4}{2}$
8 fourths
$8 \times \frac{1}{4} = \frac{8}{4}$

   These sketches will reveal that $\frac{2}{1} = \frac{4}{2} = \frac{8}{4}$

2. Have the students write the simplest name for the following

   a. $\frac{5}{5}$ = (1)
   b. $\frac{36}{4}$ = (9)
   c. $\frac{12}{2}$ = (6)

3. Have the students write five fractions which are names for each whole number. Sample answers are given.

   a. 2 $\left(\frac{2}{1}, \frac{4}{2}, \frac{6}{3}, \frac{8}{4}, \frac{200}{100}\right)$
   b. 3 $\left(\frac{3}{1}, \frac{6}{2}, \frac{15}{5}, \frac{24}{8}, \frac{150}{50}\right)$

Content

FRACTIONAL NUMBERS

Equivalent Fractions
Simplest Form

Objectives

Given a model of a fractional number such as $\frac{2}{8}$, the student can

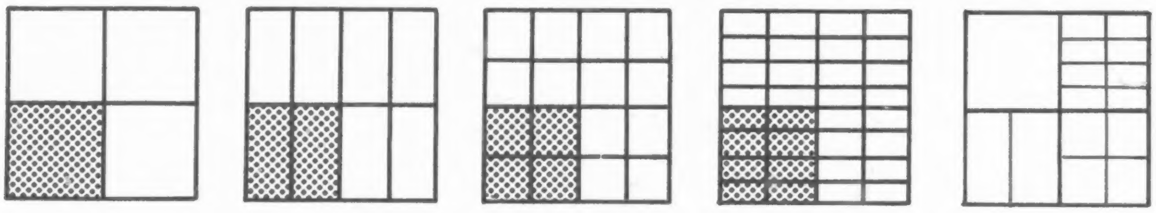
1. write a set of equivalent fractions; and
2. rename the given number in its simplest form.

ACTIVITIES

1. To illustrate equivalent fractions you may have students do the following:

   a. Prepare a wall chart or bulletin board.

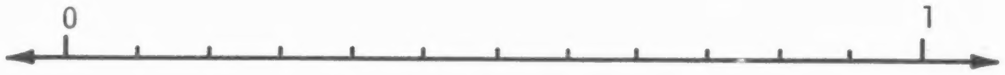
EQUIVALENT FRACTIONS: FRACTIONS THAT NAME THE SAME NUMBER



$\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$ $\frac{8}{32}$ $\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32}$

   b. Use sand and a measuring cup to illustrate $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

   c. With the help of the number line below, have students write fractions equivalent to $\frac{1}{3}$ and $\frac{2}{3}$.



$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

$\frac{0}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

$\frac{0}{12}$ $\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{7}{12}$ $\frac{8}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ $\frac{11}{12}$ $\frac{12}{12}$

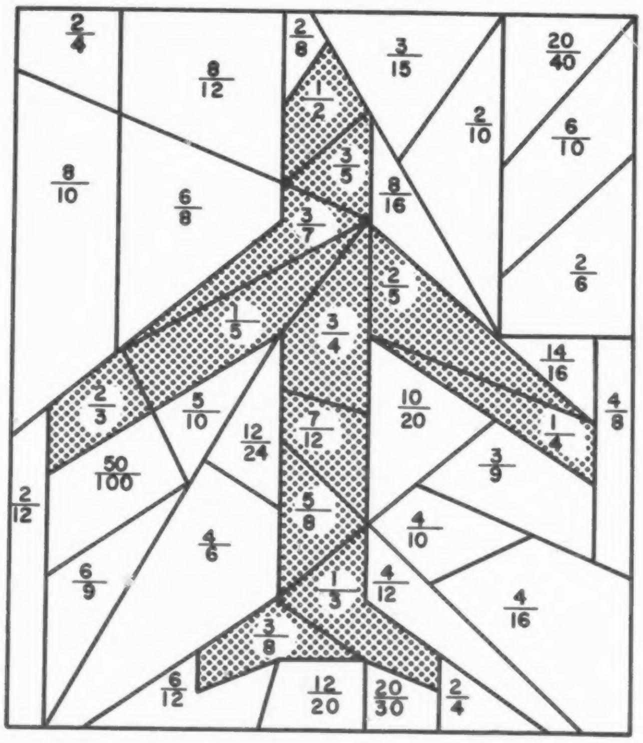
$\left(\frac{1}{3} = \frac{2}{6} = \frac{4}{12}\right)$ $\left(\frac{2}{3} = \frac{4}{6} = \frac{8}{12}\right)$

From the number line above how many names for one can you find?

$\left(\frac{3}{3}, \frac{6}{6}, \frac{12}{12}\right)$

(Continued on next page)

2. Give the class a drawing such as the one below. Have your students shade those regions in which the fractions named are in simplest form. What is the resulting picture?


2/4
8/12
2/8
3/15
20/40
1/2
2/10
6/10
8/10
6/8
3/5
8/16
3/7
2/6
1/5
3/4
2/5
14/16
2/3
5/10
10/20
4/8
12/24
7/12
1/4
50/100
3/9
2/12
5/8
4/6
4/10
6/9
1/3
4/12
4/16
3/8
6/12
12/20
20/30
2/4

Artistic students may enjoy creating some of these puzzles for the class to use.

Content

FRACTIONAL NUMBERS

Order
$<, =, >$

Objectives

Given two fractions, the student can

1. write $<$, $=$, or $>$ between them and
2. explain its meaning, for example, $\frac{1}{2} > \frac{1}{3}$ means one half is greater than one third.

ACTIVITIES

1. Write the following fractions on the chalkboard and ask the students to tell which are equivalent. If they have difficulty, ask them to give suggestions on how to determine whether or not two fractional numbers are equivalent. Hopefully, someone will suggest writing them with common denominators.

   a. $\frac{1}{2}, \frac{1}{3}$ (Since $\frac{6}{12} \neq \frac{4}{12}$, $\frac{1}{2}$ and $\frac{1}{3}$ are not equivalent)

   b. $\frac{2}{5}, \frac{4}{10}$ (Since $\frac{4}{10} = \frac{4}{10}$, $\frac{2}{5}$ and $\frac{4}{10}$ are equivalent)

   c. $\frac{7}{8}, \frac{2}{3}$ (Since $\frac{21}{24} \neq \frac{16}{24}$, $\frac{7}{8}$ and $\frac{2}{3}$ are not equivalent)

   Another way to compare two fractional numbers is to locate them on a number line. The number farther to the right will be the greater.

2. After the students have had experiences with activities like the one above you might lead them to discover that a shorter way to compare fractional numbers is to examine their cross products. For example, in the fractions $\frac{2}{3}$ and $\frac{3}{4}$, (2×4) and (3×3) are called cross products and since $(2 \times 4) < (3 \times 3)$, $\frac{2}{3} < \frac{3}{4}$. Likewise, $\frac{2}{5} = \frac{4}{10}$, since $(2 \times 10) = (5 \times 4)$; and $\frac{7}{8} > \frac{2}{3}$, since $(7 \times 3) > (8 \times 2)$.

   Have your students insert the correct symbol $<$, $=$, or $>$ between each pair of fractions by using the cross products method.

   a. $\frac{4}{5} \textcircled{$>$} \frac{2}{3}$

   b. $\frac{5}{4} \textcircled{$<$} \frac{9}{2}$

Content

RATIONAL NUMBERS

Fractions
Greater Than One
Mixed Numerals

Objective

Given a fraction representing a number greater than one, the student can rename it as a mixed numeral, and vice versa.

ACTIVITIES

1. Have your students work in pairs or small groups using fraction kits, number lines, or their own drawings to determine different names for such numbers as six and one half, eleven eighths, or five thirds. Ask them to record the results of these explorations. After several names for each number have been identified by each group, some of the students may record their work on the chalk board to explain how they arrived at their particular answers.

2. Discuss with your students the renaming of $6\frac{1}{2}$ as a fraction. First rewrite the number as $6 + \frac{1}{2}$. Then rename the 6 as $\frac{12}{2}$. This gives $\frac{12}{2} + \frac{1}{2}$ which equals $\frac{13}{2}$.

3. Let one half the members of a small group write a series of mixed numerals for the other members to rewrite as fractions. Ask them to discuss, within their group, the process each used to make the change. Compare the results to determine how many different ways were used.

Content

RATIONAL NUMBERS

Decimal
Tenths
Hundredths

Objectives

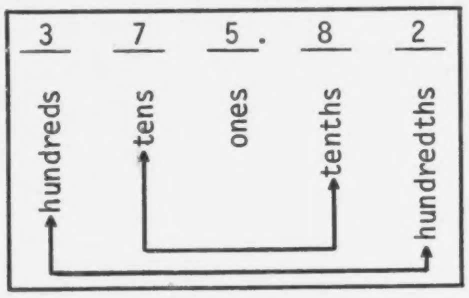
Given fractions having denominators of 10 or 100, the student can
1. identify,
2. read and
3. write

the decimal name for these numbers.

ACTIVITIES

1. To develop an awareness of decimals and some ways in which they are used - have pupils observe the digits on an automobile odometer or the gauge on a gasoline pump, or the markings on a rain gauge. Encourage discussion about what the digits on the odometer, the gasoline pump gauge and the rain gauge indicate. Be sure that the discussion brings out what happens after the nine appears in the tenths place. Let pupils offer an explanation. Ask them to choose concrete materials and/or diagrams to illustrate the division of the number one into tenths. Some materials which they might use are fraction kits, a hundreds board with strips equalling .1, a number line, or number rods. Remind pupils that there are many ways of recording the names for a number.

2. Develop with the students a chart beginning with the one's place, then naming the tens and hundreds to the left, and the tenths and hundredths to the right. Help them to see that once the ones place is located the other places in the numeral are related in pairs. For example, the ten digit one place to the left of the ones digit corresponds to the tenths digit which is one place to the right of the ones digit as shown in the sketch. The hundreds digit can be seen to correspond to the hundredths digit. By picturing this symmetry that exists the pupil should be able to better understand the rationale for the names given each of the places in this numeration system. Try to get the students to explain that the decimal separates the ones and the tenths places. The word "and" separates the whole number name from the fraction name.



3. Suggest that pupils find instances where decimals involving tenths and hundredths are frequently used. Some examples are grocery store meat departments, cash registers, newspapers, our money system and the Dewey Decimal system of the library.

Content

RATIONAL NUMBERS

Fractions
Decimals
Mixed Numerals

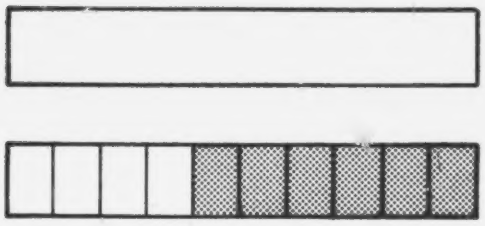
Objective

Given a set of numerals such as $\frac{1}{3}$, $1\frac{1}{2}$, .23, the student can classify them as fractions, decimals or mixed numerals.

ACTIVITIES

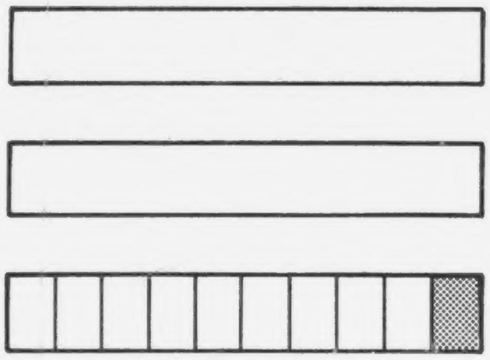
1. Provide your students with many strips of paper as sketched below. Mark some to indicate tenths. Shade the parts which are not to be counted. Distribute several of these to each student and ask him to name the numbers represented. Have him name these numbers both as a decimal and a mixed numeral.

a.



(1.4, $1\frac{4}{10}$)

b.



(2.9, $2\frac{9}{10}$)

2. Provide your students with exercises like these to review the relationships that exists between fractions, decimals, and mixed numerals.

Write mixed numerals

a. $5 + \frac{3}{7}$ b. $\frac{8}{2} + \frac{3}{2}$ c. $\frac{3}{5} + 6$ d. 27.4

Write fractions

e. $9\frac{3}{5}$ f. $11\frac{5}{11}$ g. .3 h. .07

Write decimals

i. $5\frac{5}{10}$ j. $\frac{7}{10}$

Content

RATIONAL NUMBERS

Expanded Notation

Objective

Given a number such as 24.73, the student can write its expanded numeral in the following way:

$24.73 = (2 \times 10) + (4 \times 1) + (7 \times \frac{1}{10}) + (3 \times \frac{1}{100})$

ACTIVITIES

A series of exercises like these might be used to help lead the student to an understanding of the use of expanded notation with decimals.

1. Write the missing word and numeral:

   a. 245.67: The 5 is in the (ones) place and means (5 x 1)

   b. 356.78: The 5 is in the (tens) place and means (5 x 10)

   c. 234.56: The 5 is in the (tenths) place and means $(5 \times \frac{1}{10})$

   d. 567.89: The 5 is in the (hundreds) place and means (5 x 100)

   e. 123.45: The 5 is in the (hundredths) place and means $(5 \times \frac{1}{100})$

2. Complete:

   a. $2.57 = 2 + \frac{\boxed{5}}{10} + \frac{7}{100}$

   b. $35.46 = 35 + \frac{4}{10} + \frac{\boxed{6}}{100}$

   c. $356.78 = 356 + \frac{\boxed{7}}{10} + \frac{\langle 8 \rangle}{100}$

3. Write an expanded numeral for each number.

   a. 4.5 $(4 \times 1) + (5 \times \frac{1}{10})$

   b. 5.67 $(5 \times 1) + (6 \times \frac{1}{10}) + (7 \times \frac{1}{100})$

   c. 23.45 $(2 \times 10) + (3 \times 1) + (4 \times \frac{1}{10}) + (5 \times \frac{1}{100})$

Content

RATIONAL NUMBERS

Names for Numbers

Objective

Given a rational number such as $2\frac{1}{2}$, the student can write many different names for the number, for example, 2.5, $\frac{5}{2}$, $\frac{10}{4}$.

ACTIVITIES

1. Ask students to locate information containing references to numbers in books, newspapers, and periodicals. Have children read these statements aloud while their classmates write the names for these numbers using Arabic numerals. Divide the class into groups of students. Have each group select several of these numbers to express by many different names. Each group might then scramble its number names for another group to try to unscramble. A group or team which comes up with a number name that another team incorrectly interpreted would get a point for stumping them. If a team creates an incorrect name which is detected by the decoding team, the students doing the unscrambling would receive a point.

2. Another contest you may wish to use involves having someone name a number, which is then to be named in many different ways by the students in a given period of time. For example, you may call out "12" and allow the class 4 minutes to work. They could write:

$$\frac{24}{2}, (10 + 2), (14 - 2), (36 \times \frac{1}{3}), (4 \times 4 - 4), \text{etc.}$$

When time is up, have the students exchange papers and judge the correctness of the answers. The student with the most correct answers is the winner.

Content

NATURAL NUMBERS

Roman Numerals
(I,V,X,L,C,D,M)

Objective

Given an Arabic numeral such as 104, the student can write the corresponding Roman numeral, CIV.

ACTIVITIES

1. Have your students complete the following sentences by converting the Arabic numerals into Roman numerals.

   a. 48 = 40 + 8 = XL + VIII = (XLVIII)
   b. 13 = 10 + 3 = (X) + (III) = (XIII)
   c. 104 = 100 + 4 = (C) + (IV) = (CIV)
   d. 333 = 300 + 30 + 3 = (CCC) + (XXX) + (III) = (CCCXXXIII)
   e. 94 = 90 + 4 = (XC) + (IV) = XCIV
   f. 179 = 100 + 70 + 9 = (C) + (LXX) + (IX) = (CLXXIX)

2. Have students make a clock face, a calendar, or a number line using Roman numerals instead of Arabic numerals.

3. As you study other subjects during the day have the students write Roman numerals whenever a number occurs in the discussion. For example in social studies you might have them write Roman numerals for these dates.

   a. 1492 (MCDXCII)
   b. 1776 (MDCCLXXVI)
   c. 1941 (MCMXLI)

Content

NATURAL NUMBERS

Roman Numerals
(I,V,X,L,C,D,M)

Objective

Given a Roman numeral such as MCCXIV, the student can write the Arabic numeral 1,214.

ACTIVITIES

1. After reviewing or introducing the basic Roman numerals, through M, have the students identify the way numerals are formed in this system of numeration. In particular call their attention to the subtractive property of this system which is illustrated in several of the examples below.

   Using the following information, complete these sentences:
   I = 1, V = 5, X = 10, L = 50, C = 100, M = 1000

   a. II = 1 + 1 = 2
   b. IV = 5 - 1 = 4
   c. VI = 5 + 1 = 6
   d. XIV = 10 + (5 - 1) = 14
   e. XIX = 10 + (10 - 1) = 19
   f. XX = 10 + 10 = 20
   g. MXL = 100 + (50 - 10) = 40
   h. LX = 50 + 10 = 60
   i. XC = 100 - 10 = 90
   j. CX = 100 + 10 = 110

2. Have your students write Arabic numerals for each of these and state the pattern that is developed.

   a. IX (9)
   b. XLIX (49)
   c. XCIX (99)
   d. CLIX (159)
   e. CCXXIX (229)
   f. CCCIX (309)

   (The pattern developed may be found in the differences of successive numbers. These difference are 40, 50, 60, 70, and 80.)

3. Use Arabic numerals to show how these statements are true:

   a. XLVII + XXXVIII = LXXXV (47 + 38 = 85)
   b. CCLV - CXXIX = CXXVI (255 - 129 = 126)
   c. (MDCCXXVIII) ÷ (XII) = CLXIV (1728 ÷ 12 = 144)
   d. (XXIV) x (LXIX) = MDCLVI (24 x 69 = 1656)

Content

WHOLE NUMBERS

Period
Place Value
Twelve-Digit Numerals

Objectives

Given a base ten numeral, the student can
1. identify and
2. name
the period for each group of three digits and the place value for each of the digits.

ACTIVITIES

1. Draw a diagram similar to the one below on the chalk board or overhead projector. Write in the headings, "ones," "thousands," etc. Ask your students to help you complete the chart.

	Billions			Millions			Thousands			Ones		
	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
a.	7	5	4	6	8	2	1	0	3	2	9	5
b.			3	6	9	8	2	1	4	7	2	6

Write some numerals in the chart and ask such questions as:

How many billions (thousands, millions) are in a?

How many thousands (ones, billions) does b contain?

*2. As a special assignment, have a student explore periods larger than billions.

Content

WHOLE NUMBERS

Word Names
Twelve-Digit Numerals

Objectives

Given a twelve-digit numeral, the student can

1. read it and
2. write its word name.

ACTIVITIES

1. Have your students bring information to class relating to large numbers. Have them convert from word names to Arabic numerals and from Arabic numerals to word names. The following examples illustrate what they might bring.

   a. The Gemini 6 and Gemini 7 space ships were the first to rendezvous in space. Gemini 7 set the record for the longest distance traveled in space up to that time: 5,129,400 miles. (five million, one hundred twenty-nine thousand, four hundred miles)

   b. In 1965 Reader's Digest was the leading U. S. magazine with a circulation of fifteen million, nine hundred seventy-two thousand, seven hundred thirty-two. (15,972,732) TV Guide was second with 10,546,443. (ten million, five hundred forty-six thousand, four hundred forty-three)

   c. During the first six months of a recent year the American people flew about 39,366,633,000 miles on commercial airlines. (thirty-nine billion, three hundred sixty-six million, six hundred thirty-three thousand) This represents a distance of about one million, five hundred sixty thousand times around the earth at the equator! (1,560,000)

   *d. The nearest star beyond our sun is Proxima Centauri; it is about fifty trillion miles away. (50,000,000,000,000)

   *e. It is estimated that the earth weighs about 6,586,242,500,000,000,000,000 tons!!! Wow, does it have a weight problem! (six sextillion, five hundred eighty-six quintillion, two hundred forty-two quadrillion, five hundred trillion)

2. For students having difficulty reading large numbers, have them make a chart like the one below, write numerals in the chart, and then try to read them.

Billions	Millions	Thousands	Ones

Content

WHOLE NUMBERS

Exponential Notation

Objective

Given a number such as 3^5, the student can rename it as (3 x 3 x 3 x 3 x 3) or 243.

ACTIVITIES

1. The following activities will reveal how well a student understands exponential notation as an example of mathematical shorthand.

   a. 3^2 = 3 x 3 = 9

   b. 3^5 = 3 x 3 x 3 x 3 x 3 = 243

   c. 5^3 = 5 x 5 x 5 = 125

   d. 2^6 = 2 x 2 x 2 x 2 x 2 x 2 = 64

   e. 10^3 = 10 x 10 x 10 = 1,000

   f. 1^8 = 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 = 1

   g. 2^3 = 2 x 2 x 2 = 8

2. Have your students put their knowledge of exponential notation to work by asking such questions as:

   a. Which is greater, 2^3 or 3^2? ($2^3 = 8$, $3^2 = 9$, so 3^2 is greater)

   b. Which is less, 3^4 or 4^3? ($3^4 = 81$, $4^3 = 64$, so 4^3 is less)

3. To run full cycle, you might give the students the factorization of a number and ask them to express this using exponents as illustrated below.

   Use exponential notation to write

   a. 7 x 7 x 7 x 7 (7^4)

   b. 10 x 10 x 10 x 10 x 10 (10^5)

Content

WHOLE NUMBERS

Prime Factorization
Exponential Notation

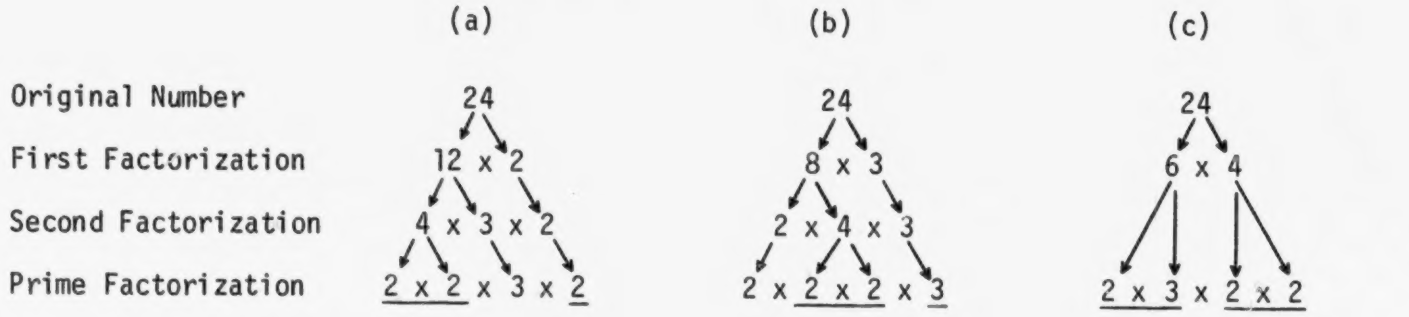
Objectives

Given a composite number, the student can

1. write the prime factorization and
2. use exponents to more concisely express this product.

ACTIVITIES

1. This activity shows that the prime factors of numbers can be found in several ways. Give the class a number such as 24 and ask them to determine its prime factorization. The following factor trees illustrate three of the ways in which the prime factors of 24 may be found.


(a)
(b)
(c)
Original Number
First Factorization
Second Factorization
Prime Factorization
24
12 x 2
4 x 3 x 2
2 x 2 x 3 x 2
24
8 x 3
2 x 4 x 3
2 x 2 x 2 x 3
24
6 x 4
2 x 3 x 2 x 2

What is the prime factorization of 24 found in example (a)?	(2 x 2 x 3 x 2 = 2^3 x 3)
What is the prime factorization of 24 found in example (b)?	(2 x 2 x 2 x 3 = 2^3 x 3)
What is the prime factorization of 24 as found in example (c)?	(2 x 3 x 2 x 2 = 2^3 x 3)

Even though the first factorizations were different in the examples above, what can be concluded about the three prime factorizations of 24 illustrated in this example?
(Except for the order of the factors they are the same, 2^3 x 3)

Content

WHOLE NUMBERS

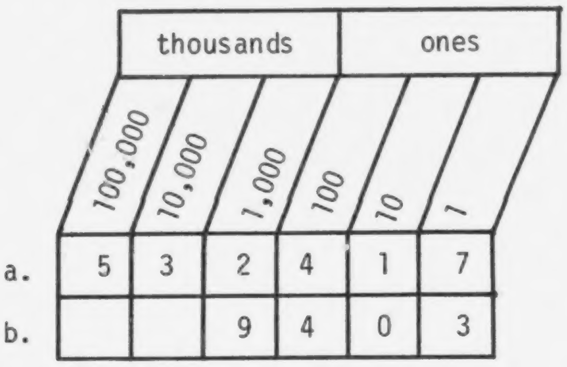
Expanded Notation
Exponential Notation

Objective

Given a number such as 275, the student can write it in expanded form such as
$(2 \times 10^2) + (7 \times 10^1) + (5 \times 1)$
using exponential notation.

ACTIVITIES

1. Have the students work with a chart similar to the one below which shows the place value of each digit.



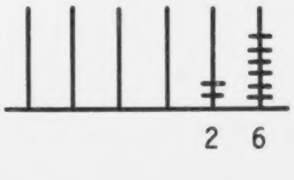
Instruct the students to first write a numeral on the chart, and then write the expanded notation as shown below.

a. 532,417 = (5 x 100,000) + (3 x 10,000) + (2 x 1,000) + (4 x 100) + (1 x 10) + (7 x 1)
= $(5 \times 10^5) + (3 \times 10^4) + (2 \times 10^3) + (4 \times 10^2) + (1 \times 10^1) + (7 \times 1)$

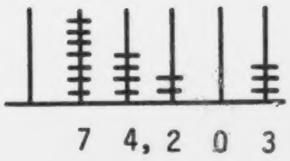
b. 9,403 = (9 x 1,000) + (4 x 100) + (0 x 10) + (3 x 1)
= $(9 \times 10^3) + (4 \times 10^2) + (0 \times 10^1) + (3 \times 1)$

2. On an abacus or sketch of an abacus represent some numbers and have students write expanded notation for them. Answers are shown to the right of the sketches.

a.



(2 x 10) + 6
$(2 \times 10^1) + (6 \times 1)$



(7 x 10,000) + (4 x 1,000) + (2 x 100) + (0 x 10) + (3 x 1)
$(7 \times 10^4) + (4 \times 10^3) + (2 \times 10^2) + (0 \times 10^1) + (3 \times 1)$

Content

NATURAL NUMBERS

Roman Numerals
I, V, X, L, C, D, M

Objective

Given either a Roman or an Arabic numeral, the student can write its equivalent in the other numeration system.

ACTIVITIES

1. Have one or several students prepare a poster or bulletin board reviewing the values of the Roman numerals: I, V, X, L, C, D, and M.

2. On the cornerstone or a metal plaque found in many public buildings the date of construction is indicated in Roman numerals. Have students find as many dates from cornerstones as possible and convert these dates to Arabic numerals.

3. Give the students some famous dates written in Roman numerals. Have them determine the Arabic equivalents and tell why the dates are important. This should relate well to your work in social studies. Here are some sample dates:

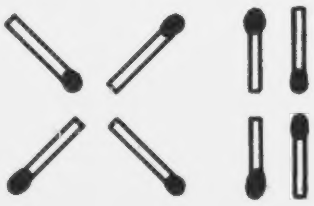
a.	MCDXCII	(1492)	d.	MCCXV	(1215)
b.	MDCVII	(1607)	e.	MCMXVII	(1917)
c.	MDCCLXXVI	(1776)	f.	MCMXLI	(1941)

4. Have some volunteers prepare a calendar for the month using Roman numerals instead of Arabic numerals.

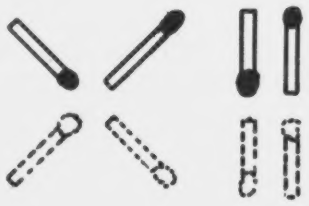
5. Ask your students to write in Roman numerals several important dates in their lives such as their birthday. For example: April 7, 1959 or 4-7-1959 would be IV-VII-MCMLIX.

6. Here is an interesting "attention getter" which illustrates the importance of distinguishing between numbers and numerals.

   a. How can eight matches be arranged to make 12?



   b. How can one half of eight matches be made to equal 7?



Content

WHOLE NUMBERS

Place Value
Non-decimal Bases
Base Two
Base Five
Base Eight
Base Twelve

Objectives

Given a set of objects, the student can
1. count them and
2. write numerals

in such non-decimal numeration systems as bases two, five, eight, and twelve.

ACTIVITIES

1. Have students "count" by writing numerals in various bases. Preparing a chart like the one below is helpful not only for counting in other bases but also for comparing numerals in several bases.

Base Ten	Base Two	Base Five	Base Eight	Base Twelve
1	1	1	1	1
2	10	2	2	2
3	11	3	3	3
4	100	4	4	4
5	101	10	5	5
6	110	11	6	6
7	111	12	7	7
8	1000	13	10	8
9	1001	14	11	9
10	1010	20	12	T
11	1011	21	13	E
12	1100	22	14	10

2. Provide the students with physical objects such as ice cream sticks, tongue depressors, dried beans, grains of corn, or counters.

   Give each of several children the same number of objects. Have them count the objects by grouping them in different ways to illustrate their understanding of different number bases.

3. Students may be interested to know that some aspects of other bases are used in special situations.

   Elements of base 60, for example, are used in the keeping of time. There are 60 seconds in a minute and 60 minutes in an hour.

   A circle which may be divided into 360 degrees has this same relationship within each degree, i.e., 60 seconds of angle equal 1 minute and 60 minutes equal 1 degree.

   Base two, the binary system of numeration, has practical application in electronic computers. Grouping by twos is found in a familiar system of liquid measure:
   2 cups = 1 pint, 2 pints = 1 quart, 2 quarts = 1 half gallon, and 2 half gallons = 1 gallon.

(Continued on next page)

The duodecimal Society of America, founded in 1944, has advocated adoption of the base twelve numeration system. They cite many examples to support their recommendation. Many commonly occurring fractions have a simpler form in base twelve than in base ten. This is true because 12 has six divisors - 1, 2, 3, 4, 6, and 12 - while 10 has only 4 divisors - 1, 2, 5, and 10. Other examples for considering base twelve over base ten include items sold by the dozen or gross, twelve months in a year, and 12 inches in a foot. Have your students suggest and discuss some disadvantages in converting from the decimal to the duodecimal system.

4. Some students may enjoy making up a number system by including symbols and names of numbers for an imaginary place. They can write a story including number facts about their fantasy land.

NUMBERS G-8

Content

WHOLE NUMBERS

- Non-decimal Bases
  - Base Two
  - Base Five
- Expanded Notation
- Exponential Notation
- Place Value

Objectives

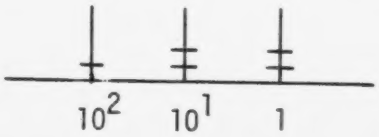
Given a number in some non-decimal numeration system such as base two or base five, the student can write

1. the expanded numeral and
2. the equivalent decimal numeral.

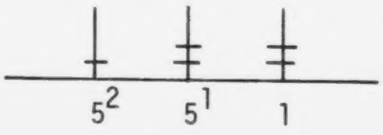
ACTIVITIES

1. Have a student use an abacus or sketch of one to illustrate 122. In the decimal system, base ten, this number can be written as

   $(1 \times 10^2) + (2 \times 10^1) + (2 \times 1)$

   Now have your students illustrate 122_{five} on the abacus. Ask them what has changed. (The value of each place in the numeral is now a power of five rather than ten.)

   Have them determine what 122_{five} represents in the decimal system.

$$\begin{aligned} 122_{five} &= (1 \times 5^2) + (2 \times 5^1) + (2 \times 1) \\ &= (1 \times 25) + (2 \times 5) + (2 \times 1) \\ &= 25 + 10 + 2 \\ &= 37 \end{aligned}$$

   Similar questions involving other bases can be presented. The students, working with counters or multibase blocks, might enjoy the challenge of converting from one non-decimal system to another, for example:

   Express 12_{five} in base two. (111_2)

Content

FRACTIONAL NUMBERS

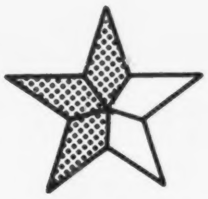
Fractions
Naming
Writing

Objectives

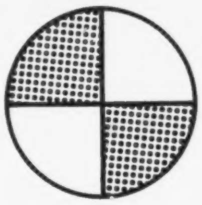
Given a model of a fractional number such as three fourths, the student can
1. name and
2. write
a fraction for this model.

ACTIVITIES

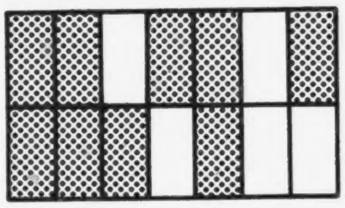
1. Give your students models such as the following to use in identifying fractions. What part of each object below is shaded?

a.


$\left(\frac{3}{5}\right)$

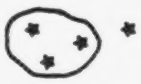
b.


$\left(\frac{2}{4} \text{ or } \frac{1}{2}\right)$

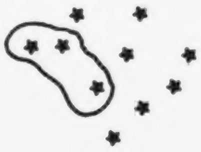
c.


$\left(\frac{9}{14}\right)$

What part of each set is enclosed?

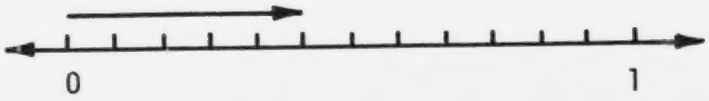
d.


$\left(\frac{3}{4}\right)$

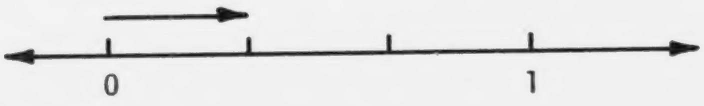
e.


$\left(\frac{3}{10}\right)$

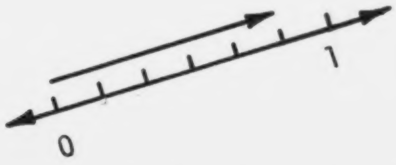
What fractional number does each directed line segment (arrow) represent?

f.


$\left(\frac{5}{12}\right)$

g.


$\left(\frac{1}{3}\right)$

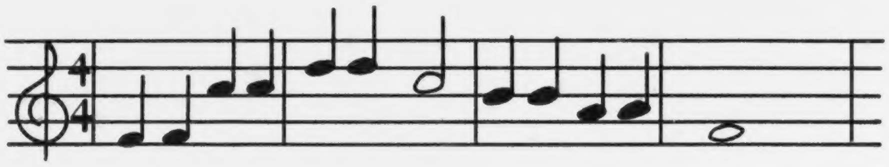
h.


$\left(\frac{5}{6}\right)$

2. Ask questions relating to physical situations involving the students such as:
   a. What fractional number indicates the part of our class absent today?
   b. What part of the class is boys?

(Continued on next page)

3. Many students who read music may not have associated mathematics with it. One example of mathematics in music is found in the symbols used for writing musical notes. Here is a sample.



Each type of note has its own value as indicated in the following chart.

Symbol	Name	Value
𝅝	whole note	4 beats
𝅗𝅥	half note	2 beats
♩	quarter note	1 beat

Music is written in sections called measures. Each measure contains the same number of beats. Since the music above is written in $\frac{4}{4}$ time, each measure contains four beats. Ask the class to write several measures in $\frac{4}{4}$ time using any arrangement of whole, half, and quarter notes. Then have them check to see if each measure contains the same number of beats. This activity will involve the addition of fractions.

Ask a student who can read music to bring a musical composition to class and explain how the "time" is counted.

You may wish to borrow some sheet music from the band director to examine with your class.

Content

FRACTIONAL NUMBERS

Multiplication Property of 1

Objectives

Given the number 1, the student

1. recognizes its special property in multiplication of fractional numbers, and
2. can name it in many ways such as $\frac{2}{2}$, $\frac{4}{4}$, $\frac{30}{30}$, etc.

ACTIVITIES

1. Direct your students to complete the following sentences to test their familiarity with the multiplication property of 1.

a. $1 \times \underline{\ 14\ } = 14$

b. $1 \times 0 = \underline{\ 0\ }$

c. $\frac{2}{3} \times 1 = \underline{\ \frac{2}{3}\ }$

d. $\frac{3}{4} \times \underline{\ 1\ } = \frac{3}{4}$

e. $\frac{5}{2} \times \frac{1}{1} = \underline{\ \frac{5}{2}\ }$

f. $1 \times \frac{7}{12} = \underline{\ \frac{7}{12}\ }$

g. $\underline{\ 1\ } \times \frac{2}{5} = \frac{2}{5}$

What can be said about the function of 1 in each of the above problems? (When 1 and any other number are multiplied, the other number is always the product. Since 1 has this special property of not changing the identity of other numbers when it is used with them in multiplication, it is called the identity element for multiplication.)

Therefore, the identity element for multiplication is $\underline{1 \text{ or } \frac{1}{1}}$.

2. An important and useful extension of this multiplication property of 1 is that 1 can be named in many ways. Have the students give several of its names, for example:

$$\frac{2}{2}, \frac{3}{3}, \frac{100}{100}, \ldots$$

Content

FRACTIONAL NUMBERS

Equivalent Fractions

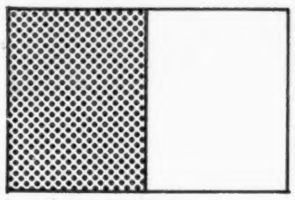
Objectives

Given a set of fractions, the student can
1. identify and
2. name

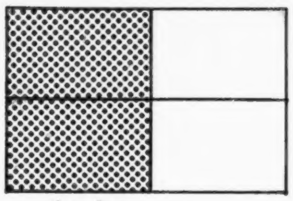
the fractions which are equivalent.

ACTIVITIES

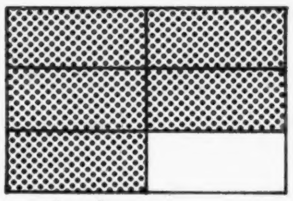
1. Give your class a set of fractions, for example, $\left\{\frac{1}{2}, \frac{2}{4}, \frac{5}{6}\right\}$. Ask them to draw a ring around those which are equivalent. An effective way to determine this is to use sketches. Have the students name the shaded part of each figure.



$\left(\frac{1}{2}\right)$



$\left(\frac{2}{4}\right)$



$\left(\frac{5}{6}\right)$

From the sketches above it is clear that $\left(\frac{1}{2} = \frac{2}{4}\right)$. $\frac{5}{6}$ names a larger part of the figure. Since $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent the students would draw a ring around each of them. $\left\{\boxed{\frac{1}{2}}, \boxed{\frac{2}{4}}, \frac{5}{6}\right\}$

2. Fractional numbers have many names, for example, one half, $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$, all name the same number.

To find another fraction which is equivalent to a given fraction, the student can multiply the original fraction by a name for one.

Give your students a set of fractions such as the following

$$\left\{\frac{14}{28}, \frac{4}{6}, \frac{2}{8}, \frac{14}{16}, \frac{6}{9}, \frac{8}{12}, \frac{16}{32}, \frac{9}{12}, \frac{10}{12}, \frac{11}{12}, \frac{20}{30}, \frac{22}{44}, \frac{35}{40}, \frac{45}{60}, \frac{12}{18}, \frac{5}{15}, \frac{75}{100}\right\}$$

and ask them to find as many equivalent fractions as they can for $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

Content

FRACTIONAL NUMBERS

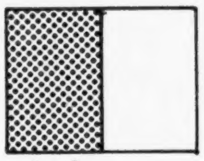
Equivalent Fractions
Including Mixed Numerals

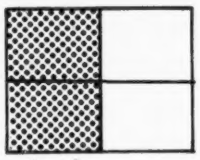
Objective

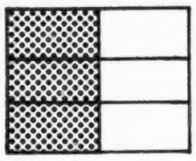
Given a fraction such as $\frac{1}{2}$, the student can write a set of fractions equivalent to it.

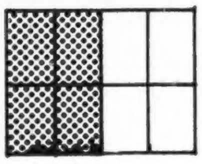
ACTIVITIES

1. Have your students use diagrams to determine several names for a fractional number. In the example below, the rectangles show four names for one half.


$\left(\frac{1}{2}\right)$


$\left(\frac{2}{4}\right)$


$\left(\frac{3}{6}\right)$


$\left(\frac{4}{8}\right)$

Now provide them with twelve, eighteen or twenty objects such as counters or dried beans to illustrate several equivalent fractions.

2. Give the class several sets of fractions and ask them to name three more for each set.

a. $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \_\_, \_\_, \_\_\right\}$

b. $\left\{\frac{3}{10}, \frac{30}{100}, \frac{15}{50}, \_\_, \_\_, \_\_\right\}$

c. $\left\{\frac{2}{3}, \frac{8}{12}, \_\_, \_\_, \_\_\right\}$

d. $\left\{\frac{4}{3}, \frac{8}{6}, \frac{12}{9}, \_\_, \_\_, \_\_\right\}$

Content

FRACTIONAL NUMBERS

Equivalent Fractions
Simplest Form
Common Factors

Objective

Given a fractional number such as $\frac{3}{12}$, the student can write its simplest name, $\frac{1}{4}$

ACTIVITIES

1. Review with your students the concept that the simplest name for a fractional number has 1 as the only common factor of its numerator and denominator. For example:

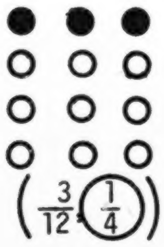
   $\frac{3}{12}$ can be written as $\frac{1 \times 3}{4 \times 3}$ or $\frac{1}{4} \times \frac{3}{3}$, and $\frac{3}{3}$ is a name for 1;

   Now substitute 1 for the $\frac{3}{3}$ and get $\frac{1}{4} \times 1$ which equals $\frac{1}{4}$

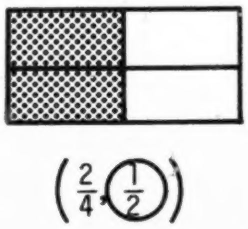
   Your students can now see that the only common factor of the numerator and denominator of $\frac{1}{4}$ is 1, and can explain why $\frac{1}{4}$ is the simplest name for $\frac{3}{12}$.

2. Diagrams will make it easier for some students to illustrate this idea. Each diagram below suggests a pair of equivalent fractions. For example, diagram a suggests $\frac{3}{12}$ equals $\frac{1}{4}$. Have the students name the pair of equivalent fractions suggested by each figure and circle the one which is in simplest form. (In diagram a, the $\frac{1}{4}$ would be circled as shown.)

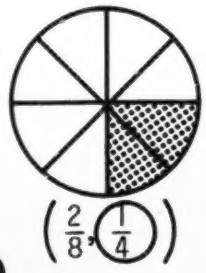
a.



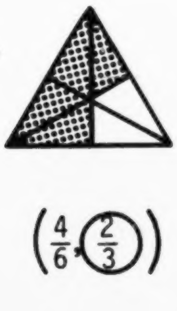
b.



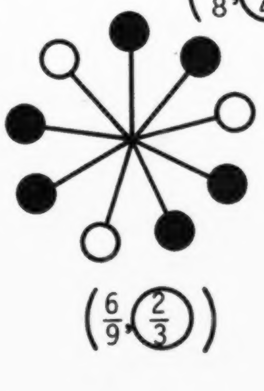
c.



d.



e.



Content

FRACTIONAL NU[illegible]S

Reciprocals

Objective

Given a fractional number such as $\frac{3}{2}$, the student can write its reciprocal, $\frac{2}{3}$.

ACTIVITIES

1. When two numbers have a product of 1, they are said to be reciprocals of each other. Knowledge of this relationship between pairs of numbers is important for work involving division of rational numbers.

   Discuss the first two examples with your students. Then have them complete the remaining exercises:

   a. $\frac{2}{3} \times \frac{3}{2} = 1$, therefore, we can say that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$ and $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$

   b. $\frac{5}{6} \times \frac{6}{5} = 1$, therefore, $\frac{6}{5}$ is the reciprocal of $\frac{5}{6}$

   c. $\frac{3}{4} \times \frac{4}{3} = \underline{1}$, therefore, $\underline{\frac{4}{3}}$ is the reciprocal of $\frac{3}{4}$

   d. $\frac{1}{5} \times \frac{5}{1} = \underline{1}$, therefore, $\underline{\frac{5}{1} \text{ or } 5}$ is the reciprocal of $\underline{\frac{1}{5}}$

   e. $\frac{7}{6} \times \frac{12}{7} = \underline{2}$, therefore, $\underline{\frac{12}{7}}$ is not the reciprocal of $\underline{\frac{7}{6}}$

   f. $\frac{8}{5} \times \frac{5}{8} = 1$, therefore, $\underline{\frac{5}{8}}$ is the reciprocal of $\underline{\frac{8}{5}}$

2. Ask your students to name the reciprocals for each of the following numbers:

   a. $\frac{3}{2}$ $\left(\frac{2}{3}\right)$

   b. $\frac{2}{7}$ $\left(\frac{7}{2}\right)$

   c. 6 $\left(\frac{1}{6}\right)$

   d. $\frac{99}{100}$ $\left(\frac{100}{99}\right)$

   e. $\frac{1}{2}$ $\left(\frac{2}{1} \text{ or } 2\right)$

   f. $\frac{8}{3}$ $\left(\frac{3}{8}\right)$

   g. 1 (The reciprocal of 1 is 1 itself because 1 x 1 = 1)

   h. 0 (0 has no reciprocal. If there were one it would be $\frac{1}{0}$; but division by zero is impossible - can you show why?)

Content

FRACTIONAL NUMBERS

Mixed Numerals
From Model to Numeral

Objectives

Given a model of a mixed number, the student can

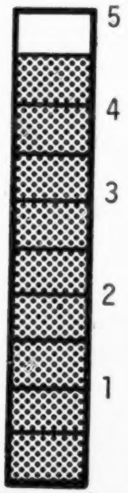
1. identify,
2. read, and
3. write

the fraction and the mixed numeral associated with the model.

ACTIVITIES

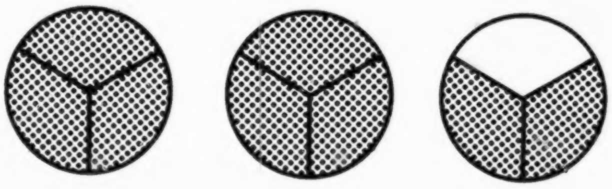
1. What number does each of the following sketches represent? In each example, write the number both as a fraction and a mixed numeral.

a.



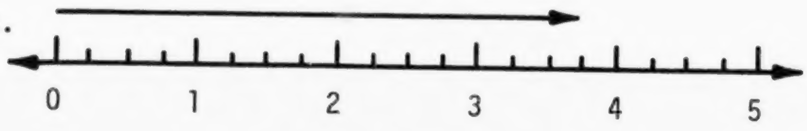
$\left(\frac{9}{2}, 4\frac{1}{2}\right)$

b.



$\left(\frac{8}{3}, 2\frac{2}{3}\right)$

c.



$\left(\frac{15}{4}, 3\frac{3}{4}\right)$

Content

FRACTIONAL NUMBERS

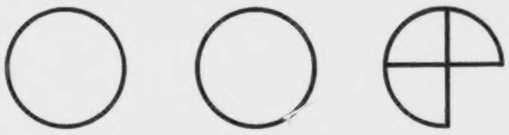
Mixed Numerals

Objective

Given a mixed numeral, the student can write it as a fraction; or given a fraction greater than one, the student can write it as a mixed numeral.

ACTIVITIES

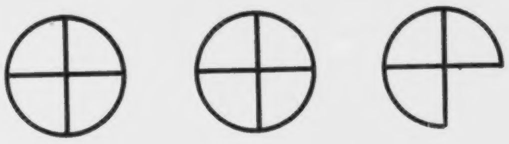
1. To refresh the students' understanding of renaming a mixed number as a fraction, a physical demonstration may be performed using discs.



These discs illustrate

$\left(2 + \frac{3}{4}\right)$ or $2\frac{3}{4}$

Cut the two whole discs into fourths. This can be quickly done if the backs of the discs are marked into fourths.



As students count the pieces, they will find that there are 11 fourths.

$\frac{4}{4} + \frac{4}{4} + \frac{3}{4} = \frac{11}{4}$

They can now conclude that $2\frac{3}{4} = \frac{11}{4}$

This activity can be effectively demonstrated on a flannel board, a magnetic board, a bulletin board or with an overhead projector; or be performed by individual students with discs at their desks.

2. To demonstrate the writing of a fraction as a mixed numeral use the reverse of the above activity. First arrange the 11 fourths randomly. Then have students arrange the fourths into wholes demonstrating that $\frac{11}{4} = 2\frac{3}{4}$.

3. Ask the students to write as mixed numerals:

a. $\frac{7}{4} = \underline{\left(1\frac{3}{4}\right)}$
b. $\frac{13}{5} = \underline{\left(2\frac{3}{5}\right)}$
c. $\frac{21}{5} = \underline{\left(4\frac{1}{5}\right)}$
d. $\frac{47}{10} = \underline{\left(4\frac{7}{10}\right)}$

4. Have the students rename these numbers as fractions:

a. $2\frac{1}{2} = \underline{\left(\frac{5}{2}\right)}$
b. $8\frac{2}{3} = \underline{\left(\frac{26}{3}\right)}$
c. $1\frac{3}{5} = \underline{\left(\frac{8}{5}\right)}$
d. $3\frac{7}{8} = \underline{\left(\frac{31}{8}\right)}$

5. Sometimes in subtractions, such as $6\frac{1}{7} - 3\frac{4}{7}$, it is necessary to write a mixed numeral as another mixed numeral as $6\frac{1}{7} = 5\frac{8}{7}$. This will give $5\frac{8}{7} - 3\frac{4}{7} = 2\frac{4}{7}$.

Have the class complete the following:

a. $2\frac{1}{3} = 1\frac{\textcircled{4}}{3}$
b. $8\frac{3}{4} = 7\frac{\textcircled{7}}{4}$
c. $9\frac{3}{5} = 8\frac{\textcircled{8}}{5}$
d. $5\frac{5}{12} = \textcircled{4}\frac{17}{12}$

Content

RATIONAL NUMBERS

Decimals
  Tenths
  Hundredths
  Thousandths
Place Value

Objectives

Given a fractional number with a denominator of 10, 100, or 1000 the student can
1. write and
2. read
the number as a decimal.

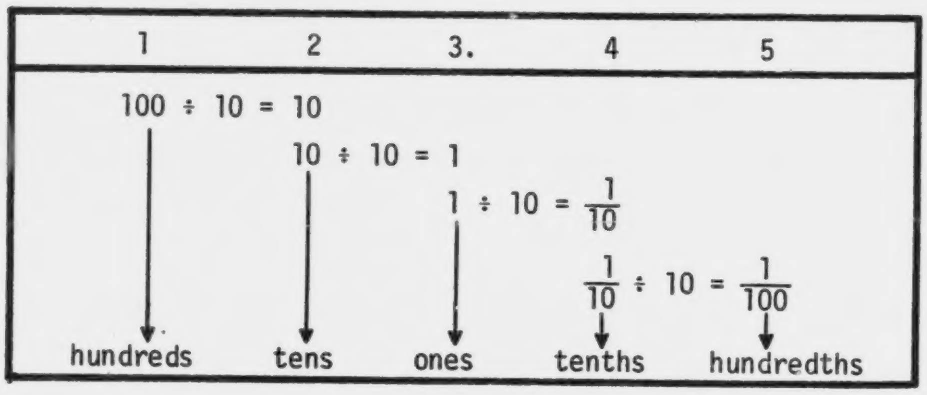
ACTIVITIES

1. A look again at place value will suggest to your students how decimals received their names. Have them closely inspect 123.45 as shown below.

Number	1	2	3.	4	5
Place Value for Each Digit	100 hundreds	10 tens	1 ones	? ?	? ?

As they look at the digits from left to right they can see a pattern in the place value. The value of each digit is one tenth of the value of the digit to its left.
Thus: $\frac{1}{10} \times 100 = 10$ and $\frac{1}{10} \times 10 = 1$.

As this pattern continues it will produce $\frac{1}{10} \times 1 = \frac{1}{10}$ which can be named tenths, and $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ which can be called hundredths. This pattern can also be interpreted with division by as shown in the example which follows.



Now have the students write the word name and decimal equivalent for each of the following:

a. $\frac{3}{10}$ (three tenths, .3)

b. $\frac{35}{100}$ (thirty-five hundredths, .35)

c. $\frac{362}{1000}$ (three hundred sixty-two thousandths, .362)

d. $\frac{3}{100}$ (three hundredths, .03)

Content

RATIONAL NUMBERS

Decimals
Expanded Notation

Objective

Given a number such as 23.74 the student can write the expanded notation in the following way:

$$23.74 = (2 \times 10) + (3 \times 1) + (7 \times \frac{1}{10}) + (4 \times \frac{1}{100}).$$

ACTIVITIES

1. Provide the students with the following chart.

	Millions (1,000,000)	Hundred Thousands (100,000)	Ten Thousands (10,000)	Thousands (1,000)	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1,000}$	Ten Thousandths $\frac{1}{10,000}$
a.			4	3	2	7	5				
b.						6	2	5	3		
c.						2	0	1	2	6	
d.							7	3	8	9	8

Have them use the chart to write the expanded notation of the four numbers which follow.

a. $43,275 = (4 \times 10,000) + (3 \times 1,000) + (2 \times 100) + (7 \times 10) + (5 \times 1)$

b. $62.53 = (6 \times 10) + (2 \times 1) + (5 \times \frac{1}{10}) + (3 \times \frac{1}{100})$

c. $20.126 = (2 \times 10) + (0 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (6 \times \frac{1}{1,000})$

d. $7.3898 = (7 \times 1) + (3 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (9 \times \frac{1}{1,000}) + (8 \times \frac{1}{10,000})$

2. Using the chart above develop the following ideas with your students.

   a. Beginning with any digit and moving to the left, the value of that digit's column or place is 10 times the value of the preceding column or place.

   b. Beginning with any digit and moving to the right, the value of that digit's column or place is one tenth of the value of the preceding column or place.

These two observations lead to the conclusion that the value of each place immediately to the left of a given place is ten times the value of the given place; and the value of each place immediately to the right of a given place is one-tenth the value of the given place. This is the effect of place value in the base ten numeration system.

Content

RATIONAL NUMBERS

Decimals
Word Names

Objectives

Given a number such as 62.53 the student can

1. read it and
2. write the word name.

ACTIVITIES

1. Have your students use the chart in Activity 1 of Numbers G-18 to read and write the word names for several numbers such as:

(a)	43,275	(Forty-three thousand two hundred seventy-five)
(b)	62.53	(Sixty-two and fifty-three hundredths)
(c)	20.126	(Twenty and one hundred twenty-six thousandths)
(d)	7.3898	(Seven and three thousand eight hundred ninety-eight ten thousandths)

Emphasize that "and" designates the decimal point when reading a number consisting of a whole number and a decimal. An alternate way of making the distinction between the whole number and the decimal is illustrated by reading 62.53 as "sixty-two point five three."

Content

RATIONAL NUMBERS

Decimals
Repeating
Terminating

Objectives

Given numbers such as $\frac{2}{3}$ or $\frac{4}{5}$, the student can

1. determine their decimal patterns and
2. identify them as either repeating or terminating decimals.

ACTIVITIES

1. Have a student look up the meanings of the words repeat and terminate. Relate these meanings to the decimals .1212 ... and .8

*To indicate the part of the decimal which does the repeating such as the 12 in .1212 ..., the notations $.12\overline{12}$ or $.\overline{12}$ are often used in place of the three dots. This repeated sequence is called the "period" of the decimal and, as in this example, may consist of more than one digit.

2. Ask the students to indicate whether the following are repeating or terminating decimals.

a.	.25	(Terminating)	e.	$.\overline{6}$	(Repeating)
b.	.272727...	(Repeating)	f.	.5	(Terminating)
c.	.125	(Terminating)	g.	$.\overline{123}$	(Repeating)
d.	.8	(Terminating)	h.	$4.\overline{3}$	(Repeating)

3. Have the students show by division whether the following fractions have repeating or terminating decimals. Useful patterns are to be found by observing the decimals obtained from numbers having the same denominators such as a and b, or d, e, and f.

a.	$\frac{1}{3}$	(.333... or $.\overline{3}$)	f.	$\frac{5}{9}$	(.555... or $.\overline{5}$)
b.	$\frac{2}{3}$	(.6666... or $.\overline{6}$)	g.	$\frac{5}{4}$	(1.25)
c.	$\frac{3}{5}$	(.600 or .6)	h.	$\frac{7}{8}$	(.875)
d.	$\frac{1}{9}$	(.111... or $.\overline{1}$)	i.	$\frac{5}{6}$	(.8333... or $.8\overline{3}$)
e.	$\frac{2}{9}$	(.222... or $.\overline{2}$)	j.	$\frac{15}{11}$	(1.3636... or $1.\overline{36}$)

Content

RATIONAL NUMBERS

Fractions
Mixed Numerals
Percents

Objective

Given a set of numerals such as $\frac{3}{4}$, .64, $3\frac{2}{3}$, 26%, the student can classify them as fractions, decimals, mixed numerals, or percents.

ACTIVITIES

1. Have your students separate the numerals below into the following groups: fractions, decimals, mixed numerals, and percents.

$\frac{2}{3}$
$3\frac{1}{5}$
.1
$\frac{5}{6}$
3.2
.01
$33\frac{1}{3}\%$
$\frac{1}{2}$

50%
16%
.64
$83\frac{1}{2}$
432%
$\frac{11}{12}$
$\frac{89}{100}$
.95

8%
.37
$.\overline{3}$
$5\frac{2}{3}$
27%
$\frac{4}{9}$
4.72
$2\frac{1}{2}\%$

fractions: $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{2}$, $\frac{11}{12}$, $\frac{89}{100}$, $\frac{4}{9}$

decimals .1, .01, .64, .95, .37, $.\overline{3}$

mixed numerals: $3\frac{1}{5}$, 3.2, $83\frac{1}{2}$, $5\frac{2}{3}$, 4.72

percents: $33\frac{1}{3}\%$, 50%, 16%, 432%, 8%, 27%, $2\frac{1}{2}\%$

Content

RATIONAL NUMBERS

Percent Notation
Decimals
Fractions

Objective

Given a number such as 47%, the student can name it as .47 and $\frac{47}{100}$.

ACTIVITIES

1. Create a chart similar to this for your students to complete.

	Fraction Simplest Form	 100 as Denominator	Decimal	Percent
a.	$\frac{47}{100}$	$\frac{47}{100}$	.47	47%
b.	$\frac{1}{10}$	$\frac{10}{100}$	.10	10%
c.	$\frac{1}{5}$	$\frac{20}{100}$	.20	20%
d.	$\frac{1}{20}$	$\frac{5}{100}$	.05	5%
e.	$\frac{1}{2}$	$\frac{50}{100}$	.50	50%
f.	$\frac{63}{1000}$	$\frac{6.3}{100}$	.063	6.3%
g.	1	$\frac{100}{100}$	1.00	100%

*2. Discuss the following notation with your students:

$$\begin{aligned} r\% &= r \times \frac{1}{100} \\ &= r \times .01 \end{aligned}$$

Now give them a problem which applies it:

47% of what is 84?

$$\begin{aligned} 47\% \times n &= 84 \\ 47 \times \frac{1}{100} \times n &= 84 \\ n &= \frac{84}{1} \times \frac{100}{47} \\ n &= 200 \end{aligned}$$

Content

INTEGERS

Positive
Negative

Objective

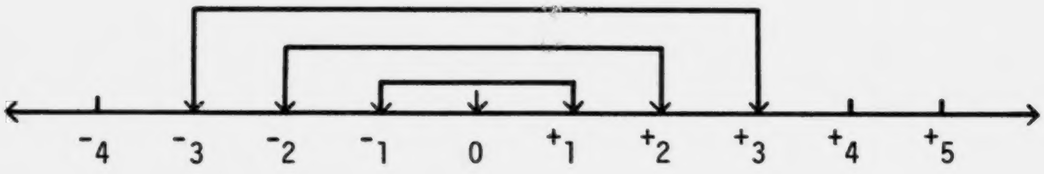
Given a numeral such as $^{-}3$, the student can name it as "negative three" or "the opposite of three".

ACTIVITIES

1. Prepare a number line showing the integers. A long line may be easily prepared using adding machine tape.

   Point to an integer such as $^{-}3$. Have students identify it as "negative three" or "the opposite of the three." On the number line have someone locate its opposite, $^{+}3$.

   The following diagram illustrates several pairs of opposites.



Content

IRRATIONAL NUMBERS

Pi (π)
Circumference
Area

Objectives

The student can
1. read,
2. write and
3. use

the symbol π to solve problems involving the circumference of a circle and the area of its region.

ACTIVITIES

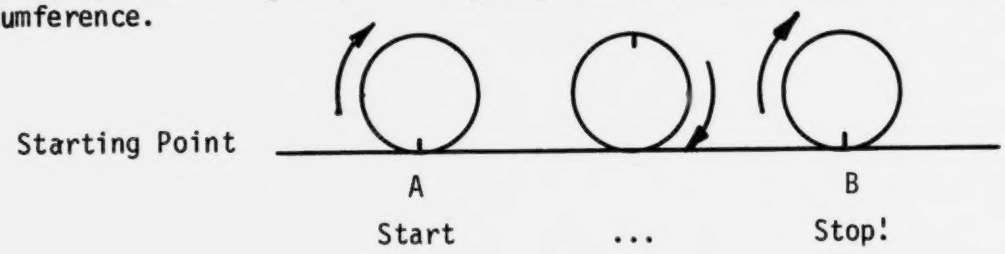
1. Have a student prepare a report on the uses of π in mathematics.

2. To understand the meaning of π, small groups of students can perform the following activity.

   Draw several circles on cardboard with diameters of different lengths. Diameters of 2, 4, 6, 8, 10 and 12 inches are suggested. Cut out the circles carefully. Mark a "starting point" on each circle.

   Draw a line segment about 50 inches long on a piece of paper, perhaps adding machine tape. Label one end of this segment point A.

   To measure the distance around each circle, place the "starting point" of the circle at point A and roll the circle along the line until the starting point again touches the segment. The distance from A to B along the line represents the length of the circle we call this number the circumference.



   Tabulate the results for each circle in a chart similar to the one below.

Diameter of Circle	Circumference of Circle	C/d (circumference divided by diameter) (Find results correct to the nearest tenth)

   The results your students get will differ, but the values in the last column should be approximately 3.1

   By comparing the circumferences of many circles to their diameters, students can draw the conclusion that π, the ratio of a circle to its diameter, is approximately 3.1 regardless of the size of the circle.

3. After developing with the class the formula for the area of a circular region, have them calculate several areas. You may have them use their experimentally developed value for π along with the two values most often used, $\frac{22}{7}$ and 3.14, to determine the area. By comparing their three answers they will see how close the three approximations are to each other.

Content

RATIONAL NUMBERS

Rounding to the Nearest
- Hundredth
- Tenth
- One
- Ten
- Hundred
- Thousand

Objective

Given a number such as 1,597.682 the student can round it to the nearest hundredth, tenth, one, ten, hundred, and thousand.

ACTIVITIES

1. Have the students complete the following statements.

   a. Rounded to the nearest hundredth 1,597.682 is (1,597.68)

   b. Rounded to the nearest hundred 1,597.682 is (1,600)

   c. 5.23 appears as 5 when rounded to the nearest (one or whole number)

   d. 5.23 appears as 5.2 when rounded to the nearest (tenth)

   e. In rounding the number 62.3794 to the nearest ten, the digit which determines whether we round to 60 or 70 is (2)

2. Tell the class to round these numbers as directed in the chart.

Round to nearest	Numbers				
	2526.321	71,111.111	8975.298	16,471.8754	3628.765
Thousand	3000	71,000	9000	16000	4000
Ten	2530	71,110	8980	16,470	3630
One	2526	71,111	8975	16,472	3629
Tenth	2526.3	71,111.1	8975.3	16,471.9	3628.8
Hundredth	2526.32	71,111.11	8975.30	16,471.88	3628.76

Note: What happens when the number you wish to round is midway between your two alternatives, for example:

Given 45, do you round to 40 or 50?

Given 450, do you round to 400 or 500?

It is generally agreed to round these numbers up to 50 and 500, but this is not a definite rule. An alternative rule suggests rounding up when the digit preceding the 5 is odd, and rounding down when the preceding digit is even. You may wish to discuss the possible reasoning behind this with your students.

NUMBERS G-26

Content

WHOLE NUMBERS

Exponential Notation
Factors

Objective

Given two numbers such as 2^3 and (2×3), the student can determine their values to show that they are not the same number.

ACTIVITIES

1. Have your students complete the following statements:

   a. In the expression 2^3, the 3 is called _(an exponent)_

   b. In the expression (2×3), the 3 is called _(a factor)_

   c. 2^3 names the number _(8)_ while (2×3) names the number _(6)_

   d. 2^3 means _$(2 \times 2 \times 2)$_ while (2×3) means _$(3 + 3)$_ or _$(2 + 2 + 2)$_

2. Ask your students to complete the following statements. Preferred answers are given although others may also be correct.

   a. 2^4 names _16_ b. 8 names _2^3_ c. 9 names _3^2_

NUMBERS G-27

Content

RATIONAL NUMBERS

Scientific Notation

Objective

Given a number such as 2,100 the student can name it using scientific notation 2.1×10^3

ACTIVITIES

1. Scientific notation for a number consists of a decimal number from 1 to 10, a multiplication symbol, and an exponential notation for a power of ten. See the example below.

$36{,}000 = 3.6 \times 10^4$

This procedure can be explained in the following way.

$$36{,}000 = 36{,}000 \times \frac{10^4}{10^4} = \frac{36{,}000}{10^4} \times 10^4 = 3.6 \times 10^4$$

Have your students try these:

a. $215{,}000 =$ _(2.15×10^5)_

b. $3{,}400{,}000 =$ _(3.4×10^6)_

c. _(620,000)_ $= 6.2 \times 10^5$

d. _(710)_ $= 7.1 \times 10^2$

Content

RATIONAL NUMBERS

Names for Numbers
  Fractions
  Mixed Numerals
  Decimals
  Percents

Objectives

Given a number such as one and one half, the student can

1. identify,
2. read and
3. write

many different names for it, for example:
$\left(2 - \frac{1}{2}\right)$, 150%, $\frac{12}{8}$, 1.5

ACTIVITIES

1. Ask your students to write the simplest Arabic numeral named by each set.

a. $\left\{ \text{V}, (4 + 1), (5 - 0), \frac{50}{100} \right\}$ (5)

b. $\left\{ (100 - 90), (7 + 3), (5 \times 2), \text{X} \right\}$ (10)

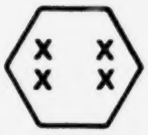
c. $\left\{ 4^2, (20 - 2^2), \text{XVI} \right\}$ (16)

d. $\left\{ (2 - \frac{1}{2}), 150\%, \frac{12}{8}, 1.5 \right\}$ $\left(\frac{3}{2}\right)$

2. Have individual students or small groups of students look up the systems of numeration used by the Romans, Greeks, Babylonians, Egyptians and Mayans. Ask them to prepare posters for a bulletin board display. It is necessary for students to memorize only the Roman numeration system.

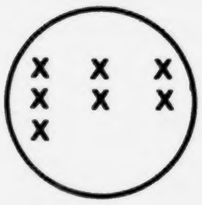
3. Challenge the students to write 8 different names for the number of objects inside each shape.

a.

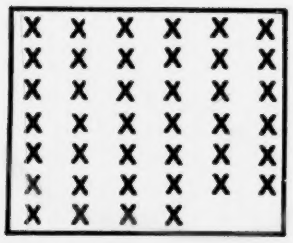


Answers will vary. Examples of correct responses are 4, 2^2, IV, (2×2), $(4 + 0)$, $(2 + 2)$, $(3 + 1)$, and $(5 - 1)$.

b.



c.



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